FRM Part I Book 3: Financial Markets and Products

Financial Markets and Products

FRM PART I BOOK 3: FINANCIAL MARKETS AND PRODUCTS

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The following is a review of the Financial Markets and Products principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Introduction (Options, Futures, and Other Derivatives)

Topic 23

Exam Focus

In this topic, we present the basic concepts of derivative securities and derivative markets. For the exam, know the basic derivative terms as well as the terms related to derivative markets. Also, be able to compute payoffs for the different derivative securities and be able to create a hedge and know how to take advantage of an arbitrage situation. As indicated by the title, this topic provides an introduction to the upcoming derivatives material.

Derivative Markets

AIM 23.1: Differentiate between an open outcry system and electronic trading.

An **open outcry system** and **electronic trading system** are different forms of trading securities (matching buyers with sellers). The open outcry system (e.g., CBOT) is the more traditional system, which involves traders actually indicating their trades through hand signals and shouting. Electronic trading does not involve an actual "physical" exchange location, but rather involves matching buyers and sellers electronically via computers (e.g., NASDAQ).

AIM 23.2: Describe the over-the-counter market and how it differs from trading on an exchange, including advantages and disadvantages.

An **over-the-counter (OTC) market** differs from a traditional exchange. It is a customized trading market which utilizes telephone and computers to make trades. This market typically involves much larger trades than traditional exchanges. The most typical OTC trade is conducted over the phone. Since terms are not specified by an "exchange," participants have more flexibility to negotiate the most mutually agreeable or attractive trade.

The OTC market is several times the size of the traditional exchange market. For example, in 2007, the OTC market was over $500 trillion, while the exchange-traded market was under $100 trillion.

Advantages of over-the-counter trading:

- Terms are not set by any exchange.
- Participants have flexibility to negotiate.
- In the event of a misunderstanding, calls are recorded.

Disadvantages of over-the-counter trading:

- OTC trading has more credit risk than exchange trading. Exchanges are organized in such a way that credit risk is eliminated.

Basics of Derivative Securities

AIM 23.3: Differentiate between options, forwards, and futures contracts.

An **option contract** is a contract that, in exchange for the option price, gives the option buyer the right, but not the obligation, to buy (sell) an asset at the exercise price from (to) the option seller (buyer) within a specified time period, or depending on the type of option, a precise date (i.e., expiration date). A call option gives the option holder the right to purchase the underlying asset by a certain specified date for a specified (in advance) price. A put option gives the option holder the right to sell the underlying asset by a selected date for a pre-selected price.

A **forward contract** is a contract that specifies the price and quantity of an asset to be delivered sometime in the future. There is no standardization for forward contracts, and these contracts are traded in the over-the-counter market. One party takes the long position, agreeing to purchase the underlying asset at a future date for a specified price, while the other party is the short, agreeing to sell the asset on that same date for that same price. Forward contracts are often used in foreign exchange situations as these contracts can be used to hedge foreign currency risk.

A **futures contract** is a more formalized, legally binding agreement to buy/sell a commodity/financial instrument in a pre-designated month in the future, at a price agreed upon today by the buyer/seller. Futures contracts are highly standardized regarding quality, quantity, delivery time, and location for each specific commodity. These contracts are typically traded on an exchange.

Professor's Note: Remember that a futures contract is an obligation/promise to actually complete a transaction, while an option is simply the right to buy/sell.

AIM 23.4: Calculate and identify option and forward contract payoffs.

Call Option Payoff

The payoff on a **call option** to the option buyer is calculated as follows:

$$C_T = \max(0, S_T - X)$$

where:
C_T = payoff on call option
S_T = stock price at maturity
X = strike price of option

The payoff to the option seller is $-C_T$ [i.e., $-\max(0, S_T - X)$]. We should note that $\max(0, S_t - X)$, where time, t, is between 0 and T, is also the payoff if the owner decides to exercise the call option early (in the case of an American option as we will discuss later).

The price paid for the call option, C_0, is referred to as the **call premium**. Thus, the profit to the option buyer is calculated as follows:

$$\text{profit} = C_T - C_0$$

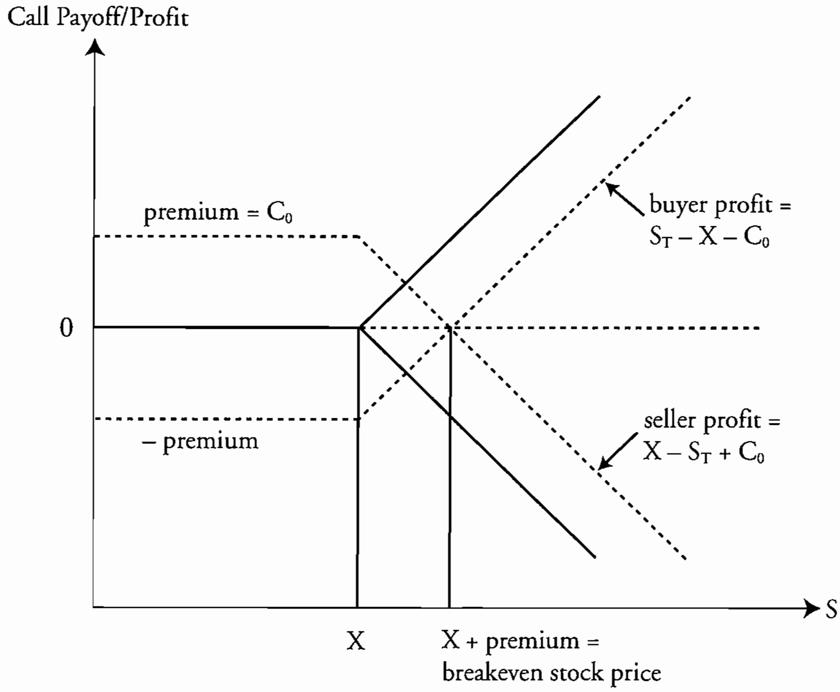
where:
C_T = payoff on call option
C_0 = call premium

Conversely, the profit to the option seller is:

$$\text{profit} = C_0 - C_T$$

Figure 1 depicts the payoff and profit for the buyer and seller of a call option.

Figure 1: Profit Diagram for a Call at Expiration



Put Option Payoff

The payoff on a **put option** is calculated as follows:

$P_T = \max(0, X - S_T)$

where:
P_T = payoff on put option
S_T = stock price at maturity
X = strike price of option

The payoff to the option seller is $-P_T$ [i.e., $-\max(0, X - S_T)$]. We should note that $\max(0, X - S_t)$, where $0 < t < T$, is also the payoff if the owner decides to exercise the put option early.

The price paid for the put option, P_0, is referred to as the **put premium**. Thus, the profit to the option buyer is calculated as follows:

$\text{profit} = P_T - P_0$

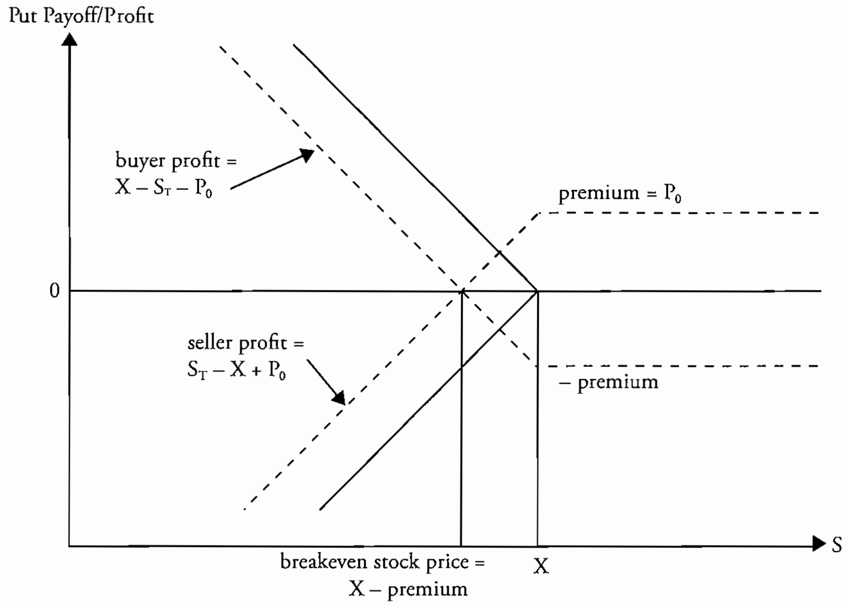
where:
P_T = payoff on put option
P_0 = put premium

The profit to the option seller is:

$\text{profit} = P_0 - P_T$

Figure 2 depicts the payoff and profit for the buyer and writer of a put option.

Figure 2: Profit Diagram for a Put at Expiration



Example: Calculating profit and payoffs from options

Compute the payoff and profit to a call buyer, a call writer, put buyer, and put writer if the strike price for both the put and the call is \$45, the stock price is \$50, the call premium is \$3.50, and the put premium is \$2.50.

Answer:

Call buyer:
payoff = $C_T = \max(0, S_T - X) = \max(0, \$50 - \$45) = \$5$
profit = $C_T - C_0 = \$5 - \$3.50 = \$1.50$

Call writer:
payoff = $-C_T = -\max(0, S_T - X) = -\max(0, \$50 - \$45) = -\$5$
profit = $C_0 - C_T = \$3.50 - \$5 = -\$1.50$

Put buyer:
payoff = $P_T = \max(0, X - S_T) = \max(0, \$45 - \$50) = \$0$
profit = $P_T - P_0 = \$0 - \$2.50 = -\$2.50$

Put writer:
payoff = $-P_T = -\max(0, X - S_T) = -\max(0, \$45 - \$50) = \$0$
profit = $P_0 - P_T = \$2.50 - \$0 = \$2.50$

Forward Contract Payoff

The payoff to a long position in a forward contract is calculated as follows:

payoff = $S_T - K$

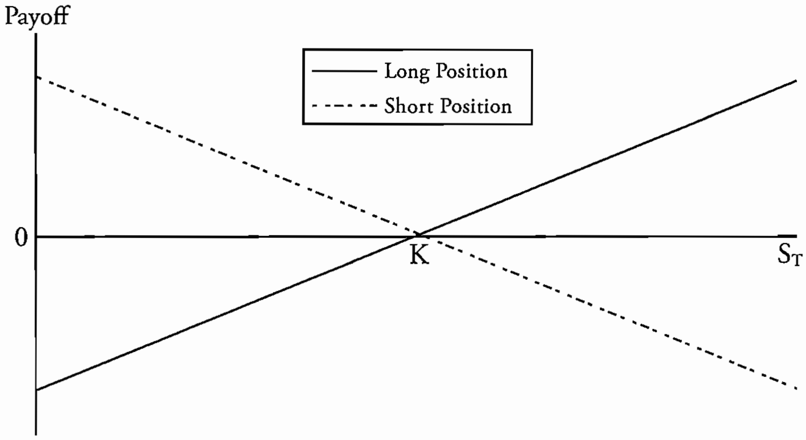
where:
S_T = spot price at maturity
K = delivery price

Conversely, the payoff to a short position in a forward contract is calculated as follows:

payoff = $K - S_T$

Figure 3 depicts the payoff for the long and short positions in a forward contract.

Figure 3: Forward Contract Payoff



Example: Calculating forward contract payoffs

Compute the payoff to the long and short positions in a forward contract given that the forward price is $25 and the spot price at maturity is $30.

Answer:

Payoff to long position:
payoff = $S_T - K$ = $30 – $25 = $5

Payoff to short position:
payoff = $K - S_T$ = $25 – $30 = –$5

Hedging Strategies

AIM 23.5: Describe, contrast, and calculate the payoffs from hedging strategies involving forward contracts and options.

Hedgers use forward contracts and options to reduce or eliminate financial exposure. An investor or business with a long exposure to an asset can hedge exposure by either entering into a short futures contract or by buying a put option. An investor or business with a short exposure to an asset can hedge exposure by either entering into a long futures contract or by buying a call option.

Hedgers use forward contracts to lock in the price of the underlying security. Forward contracts do not require an initial investment, but hedgers give up any price movement that may have had positive results in the event that the position was left unhedged. Option contracts on the other hand function as insurance for the underlying by providing the downside protection that the hedger seeks and allowing for price movement in the direction that could yield positive results. This insurance does not come without a cost, as we described earlier, since hedgers are required to pay a premium to purchase options.

Example: Hedging with a forward contract

Suppose that a company based in the United States will receive a payment of €10M in three months. The company is worried that the euro will depreciate and is contemplating using a forward contract to hedge this risk. **Compute** the following:

1. The value of the €10M in U.S. dollars at maturity given that the company hedges the exchange rate risk with a forward contract at 1.25 $/€.

2. The value of the €10M in U.S. dollars at maturity given that the company did not hedge the exchange rate risk and the spot rate at maturity is 1.2 $/€.

Answer:

1. The value at maturity for the hedged position is:
   €10,000,000 × 1.25 $/€ = $12,500,000

2. The value at maturity for the unhedged position is:
   €10,000,000 × 1.2 $/€ = $12,000,000

Example: Hedging with a put option

Suppose that an investor owns one share of ABC stock currently priced at $30. The investor is worried about the possibility of a drop in share price over the next three months and is contemplating purchasing put options to hedge this risk. **Compute** the following:

1. The profit on the unhedged position if the stock price in three months is $25.
2. The profit on the unhedged position if the stock price in three months is $35.
3. The profit for a hedged stock position if the stock price in three months is $25, the strike price on the put is $30, and the put premium is $1.50.
4. The profit for a hedged stock position if the stock price in three months is $35, the strike price on the put is $30, and the put premium is $1.50.

Answer:

1. Profit = $S_T - S_0$ = \$25 – \$30 = –\$5
2. Profit = $S_T - S_0$ = \$35 – \$30 = \$5
3. Profit = $S_T - S_0 + \max(0, X - S_T) - P_0$
   = \$25 – \$30 + max (0, \$30 – \$25) – \$1.50 = –\$1.50
4. Profit = $S_T - S_0 + \max(0, X - S_T) - P_0$
   = \$35 – \$30 + max (0, \$30 – \$35) – \$1.50 = \$3.50

Professor's Note: Notice that the max term is $5 in Case #3 and $0 in Case #4.

Speculative Strategies

AIM 23.6: Describe, contrast, and calculate the payoffs from speculative strategies involving futures and options.

Speculators have a different motivation for using derivatives than hedgers. They use derivatives to make bets on the market, while hedgers try to eliminate exposures.

The motivation for using futures in speculation is that the limited amount of initial investment creates significant **leverage**. The amount of investment required for futures is the amount of the initial margin required by the exchange. This is generally a small percentage of the notional value of the underlying, and Treasury securities can typically be posted as margin. Futures contracts can result in large gains or large losses, and contract payoffs are symmetrical.

Options also create significant leverage as investors only need to pay the option premium to purchase an option instead of the face value of the underlying. Options differ from futures in that options have asymmetrical payoffs. Gains can be quite large going long options, but losses from long option positions are limited to the option premium.

Example: Speculating with futures

An investor believes that the euro will strengthen against the dollar over the next three months and would like to take a position with a value of €250,000. He could purchase euros in the spot market at 0.80 $/€ or purchase two futures contracts at 0.83 $/€ with an initial margin of $10,000. Compute the profit from the following:

1. Purchasing euros in the spot market if the spot rate in three months is 0.85 $/€.
2. Purchasing euros in the spot market if the spot rate in three months is 0.75 $/€.
3. Purchasing the futures contract if the spot rate in three months is 0.85 $/€.
4. Purchasing the futures contract if the spot rate in three months is 0.75 $/€.

Answer:

1. Profit = €250,000 × (0.85 $/€ – 0.80 $/€) = $12,500
2. Profit = €250,000 × (0.75 $/€ – 0.80 $/€) = –$12,500
3. Profit = €250,000 × (0.85 $/€ – 0.83 $/€) = $5,000
4. Profit = €250,000 × (0.75 $/€ – 0.83 $/€) = –$20,000

A summary of these four transactions is as follows:

	Purchase Euros in Spot Market	*Purchase Long Forward Position*
Investment	$200,000	$10,000
Profit if spot at maturity = 0.85 $/€	$12,500	$5,000
Profit if spot at maturity = 0.75 $/€	–$12,500	–$20,000

Example: Speculating with options

An investor who has \$30,000 to invest believes that the price of stock XYZ will increase over the next three months. The current price of the stock is \$30. The investor could directly invest in the stock, or she could purchase 3-month call options with a strike price of \$35 for \$3. **Compute** the profit from the following:

1. Investing directly in the stock if the price of the stock is \$45 in three months.
2. Investing directly in the stock if the price of the stock is \$25 in three months.
3. Purchasing call options if the price of the stock is \$45 in three months.
4. Purchasing call options if the price of the stock is \$25 in three months.

Answer:

1. Number of stocks to purchase = \$30,000 / \$30 = 1,000
   Profit = 1,000 × (\$45 – \$30) = \$15,000
2. Profit = 1,000 × (\$25 – \$30) = –\$5,000
3. Number of call options to purchase = \$30,000 / \$3 = 10,000
   Profit = 10,000 × [max (0, \$45 – \$35) – \$3] = \$70,000
4. Profit = 10,000 × [max (0, \$25 – \$35) – \$3] = –\$30,000

Professor's Note: Since option contracts are traded in amounts of 100 options, the transactions in #3 and #4 above would entail the purchase of 100 call option contracts (i.e., 10,000 / 100 = 100).

A summary of these four transactions is as follows:

	Purchase Stock	*Purchase Call Option*
# Shares/Call option	1,000	10,000
Profit if stock at maturity = \$45	\$15,000	\$70,000
Profit if spot at maturity = \$25	–\$5,000	–\$30,000

Arbitrage Opportunities

AIM 23.7: Calculate an arbitrage payoff and describe how arbitrage opportunities are ephemeral (i.e., short-lived).

Arbitrageurs are also frequent users of derivatives. Arbitrageurs seek to earn a risk-free profit in excess of the risk-free rate through the discovery and manipulation of mispriced securities. They earn a riskless profit by entering into equivalent offsetting positions in one or more markets. Arbitrage opportunities typically do not last long as supply and demand forces will adjust prices to quickly eliminate the arbitrage situation.

Example: Arbitrage of stock trading on two exchanges

Assume stock DEF trades on the New York Stock Exchange (NYSE) and the Tokyo Stock Exchange (TSE). The stock currently trades on the NYSE for $32 and on the TSE for ¥2,880. Given the current exchange rate is 0.0105 $/¥, determine if an arbitrage profit is possible.

Answer:

Value in dollars of DEF on TSE = ¥2,880 × 0.0105 $/¥ = $30.24

Arbitrageur could purchase DEF on TSE for $30.24 and sell on NYSE for $32.

Profit per share = $32 – $30.24 = $1.76

Risk From Derivatives

AIM 23.8: Describe some of the risks that can arise from the (mis)use of derivatives.

Derivatives are versatile and can be used for hedging, arbitrage, and pure speculation. If, however, the "bet" one makes starts going in the wrong direction, the results can be catastrophic. Additionally, the risk exists that a trader with instructions to hedge a position may actually use derivates to speculate. As was shown in Book 1, this risk is known as operational risk. Controls need to be carefully established and monitored within both financial and nonfinancial corporations to prevent misuse of derivatives. Risk limits should be set, and adherence to risk limits should be monitored.

Common Terms Related to Derivatives

The following section discusses common terms associated with derivatives. Many of these terms have been mentioned earlier. Understanding these concepts will be helpful going forward as you progress through the derivatives material.

A **derivative** security is a financial security (e.g., options) whose value is derived in part from another security's characteristics or value. This other security is referred to as the underlying asset. A derivative effectively "derives" its price from some other variable.

A **market maker** is the individual that "makes a market" in a security. The market maker maintains bid and offer prices in a given security and stands ready to buy or sell lots of said security, at publicly quoted prices.

A **spot contract** is an agreement to buy/sell an asset *today*. A **forward contract** specifies the price/quantity of an asset to be delivered on or before a future pre-specified date. A **futures contract** is a legally binding agreement to buy/sell a commodity or financial instrument in a designated future month at a previously agreed upon price by the buyer/seller.

A **call option** gives its holder the right to buy a specified number of shares of the underlying security at the given strike price, on or before the option contract's expiration date. A **put option** gives the investor the right to sell a fixed number of shares at a fixed price within a given pre-specified time period. An investor may wish to have the option to sell shares of a stock at a certain price and time in order to hedge an existing investment.

An American-styled option contract can be exercised any time between issue date and expiration date. In contrast, a European-styled option contract may be exercised only on the actual expiration date. **American options** will be worth more than **European options** when the right to early exercise is valuable, and they will have equal value when it is not.

A **long position** refers to actually owning the security, while a short position is when a person sells a security he does not own. An investor taking a short position anticipates a drop in price of the security.

The exercise, or **strike price**, is the price at which the security underlying an options contract may be bought/sold.

Expiration date is the last date on which an option may be exercised.

The **bid price** is the "quoted bid," or the highest price, which a dealer is willing to pay to purchase a security. This is essentially the available price at which an investor can sell shares of stock. The **offer price** is the price at which the security is offered for sale, also known as the "asking price." The **bid-ask spread** is the difference between the ask (a.k.a. offer) price and the bid price.

Hedgers reduce their risks typically through the use of forward contracts or options. By using forward contracts, the trader is attempting to neutralize risk by fixing the price the hedger will pay/receive for the underlying asset. Option contracts, in contrast, are more of an insurance policy.

Speculators want to take a position in the market and profit from this position. Speculators are effectively betting on future price movement. When a speculator uses futures, there is a large possible gain/loss. Speculating using options is less risky since the maximum loss is the cost of the option itself.

Arbitrageurs take offsetting positions in financial instruments in order to lock in a riskless profit.

Key Concepts

1. The open outcry system is the more traditional trading system; traders actually indicate their trades through hand signals. Electronic trading involves matching up buyers and sellers electronically.

2. The over-the-counter (OTC) market is used for large trades, and a typical OTC trade is conducted over the phone. Terms are not set by an "exchange," giving traders more flexibility to negotiate mutually agreeable terms. The OTC market has more credit risk. Exchanges are organized to eliminate credit risk.

3. A forward contract is an agreement to buy or sell an asset at a pre-selected future time for a certain price.

4. A futures contract is a more formalized, legally binding agreement to buy or sell a commodity or financial asset in a pre-designated month in the future, at a price agreed upon today by the buyer/seller.

5. A call option gives its holder the right to buy a specified number of shares of the underlying security at the given strike price, on or before the option contract's expiration date, while a put option is the right to sell a fixed number of shares at a fixed price within a given pre-specified time period.

6. An American-styled option contract can be exercised any time between issue date and expiration date, while a European-styled option contract is exercised only on the actual expiration date.

7. Hedgers use derivatives to control or eliminate a financial exposure. Futures lock in the price of the underlying security and do not allow for any upside potential. Options hedge negative price movements and allow for upside potential since they have asymmetric payouts.

8. Speculators use derivatives to make bets on the market. Futures require a small initial investment, which is the initial margin requirement. Futures contracts can result in large gains or large losses as futures have a symmetrical payout function.

9. Arbitrageurs seek to earn a riskless profit through the discovery and manipulation of mispriced securities. Riskless profit is earned by entering into equivalent offsetting positions in one or more markets. Arbitrage opportunities do not last long as the act of arbitrage brings prices back into equilibrium quickly.

10. Derivatives are versatile instruments and can be used for hedging, arbitrage, and pure speculation. Controls need to be carefully established to prevent misuse of derivatives. Risk limits must be carefully established and scrupulously enforced.

11. The bid price is the "quoted bid," or the highest price for which a dealer is willing to pay to purchase a security, while the offer price is the price at which the security is offered for sale, also known as the "asking price." The bid-ask spread is the difference between the ask price and the bid price.

Concept Checkers

1. Which of the following statements is an advantage of an exchange trading system? On an exchange system:
   A. terms are not specified.
   B. trades are made in such a way as to reduce credit risk.
   C. participants have flexibility to negotiate.
   D. in the event of a misunderstanding, calls are recorded between parties.

2. Which of the following statements regarding futures contracts is most likely correct? A business with a long exposure to an asset would hedge this exposure by either entering into a:
   A. long futures contract or by buying a call option.
   B. long futures contract or by buying a put option.
   C. short futures contract or by buying a call option.
   D. short futures contract or by buying a put option.

3. Which of the following statements is least likely correct regarding the use of derivatives?
   A. Misuse of derivatives is not a very significant risk.
   B. Risk limits for derivatives should be set, and adherence to these limits should be monitored.
   C. Due to leverage inherent in derivatives, if a bet goes wrong, results can be catastrophic.
   D. There is a risk that traders may use derivatives for unintended purposes.

4. An individual that maintains bid and offer prices in a given security and stands ready to buy or sell lots of said security is a(n):
   A. hedger.
   B. arbitrageur.
   C. speculator.
   D. market maker.

5. An agreement sold over an exchange to buy/sell a commodity or financial instrument at a designated future date is known as a(n):
   A. spot contract.
   B. option contract.
   C. futures contract.
   D. forward contract.

Concept Checker Answers

1. **B** Exchanges are organized to reduce credit risk. The other answer choices are advantages of over-the-counter trading.

2. **D** A business with a long exposure to an asset would hedge the exposure by either entering into a short futures contract or by buying a put option.

3. A Misuse of derivatives can be a significant risk for firms that engage in derivatives trading.

4. **D** A market maker maintains bid and offer prices in a security and stands ready to buy or sell lots of the given security.

5. **C** A futures contract is an agreement sold on an exchange to buy/sell a commodity or financial instrument in a designated future month.

The following is a review of the Financial Markets and Products principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Mechanics of Futures Markets

Topic 24

Exam Focus

In this topic, candidates should focus on the terminology of futures markets, how futures differ from forwards, the mechanics of margin deposits, and the process of marking to market. Limit price moves, delivery options, and convergence of spot prices to futures prices are also likely exam topics. Learn the ways a futures position can be terminated prior to contract expiration and understand how cash settlement is accomplished by the final mark to market at contract expiration.

AIM 24.1: Define and describe the key features of a futures contract, including the asset, the contract price and size, delivery and limits.

AIM 24.10: Compare and contrast forward and futures contracts.

Futures contracts are exchange-traded obligations to buy or sell a certain amount of an underlying good at a specified price and date. The underlying asset varies from agricultural products to stock indices. Most futures positions are not held to take delivery of the underlying good. Instead, they are closed out or reversed prior to the settlement date.

The purchaser of a futures contract is said to have gone long or taken a **long position**, while the seller of a futures contract is said to have gone short or taken a **short position**. For each contract traded, there is a buyer and a seller. The long has contracted to buy the asset at the contract price at contract expiration, and the short has an obligation to sell at that price. Futures contracts are used by **speculators** to gain exposure to changes in the price of the asset underlying a futures contract. A **hedger**, in contrast, will use futures contracts to reduce exposure to price changes in the asset (i.e., hedge their asset price risk). An example is a wheat farmer who sells wheat futures to reduce the uncertainty about the price of wheat at harvest time.

Open interest is the total number of long positions in a given futures contract. It also equals the total number of short positions in a futures contract. An open interest of 200 would imply that there are 200 short positions in existence and 200 long positions in existence. It is possible, on any given day, for the trading volume on a contract to be higher than its open interest.

Trading Futures Contracts

To illustrate how a futures contract is created, let's use a contract on gold as an example. Each contract represents 100 troy ounces and is quoted on a per-ounce basis. Suppose an investor instructs a broker to sell one futures contract on gold with an April delivery date. At about the same time another investor instructs a broker to buy an identical futures

contract. The seller of the futures contract has a short-futures position and is obligated to sell 100 ounces of gold at the futures price at contract expiration. The buyer of the futures contract has a long futures position and is obligated to buy 100 ounces of gold at the futures price at maturity. They agree on a price of $993.60 per ounce. The two parties in this example have no idea of one another's existence because the clearinghouse (discussed in AIM 24.4) takes the opposite side of every transaction. In the futures market there is always the same number of long and short positions. This means that if a long position wins, the corresponding short position loses.

Characteristics Specified in a Futures Contract

Futures contracts are similar to forward contracts in that both allow for a transaction to take place at a future date at a price agreed upon today. The difference between the two is that forward contracts are private, customized contracts, while futures trade on an organized exchange and have terms that are highly standardized. When a new futures contract is introduced to the marketplace, the futures exchange must specify the exact terms of the contract. Futures contract characteristics specified by the exchange include the following:

- *Quality of the underlying asset.* When the underlying asset for the contract is a financial asset, such as Japanese yen, the definition of the asset is straightforward. However, when the underlying asset is a commodity, there may be different levels of quality for that good available in the marketplace (e.g., different types of wheat). The futures exchange stipulates the quality of a good that will be acceptable for settling the contract.
- *Contract size.* The contract size specifies the quantity of the asset that must be delivered to settle a futures contract (e.g., one grain contract = 5,000 bushels).
- *Delivery location.* The exchange specifies the place where delivery will take place.
- *Delivery time.* Futures contracts are referred to by the month in which delivery is to take place (e.g., a December corn contract). Some contracts are not settled by delivery but by payment in cash, based on the difference between the futures price and the market price at settlement.
- *Price quotations and tick size.* The exchange determines how the price of a contract will be quoted as well as the minimum price fluctuation for the contract, which is referred to as the **tick size.** For example, grain is quoted in dollars per bushel, and the minimum tick size is ¼ cent per bushel. Since a grain contract consists of 5,000 bushels, the minimum tick size is $12.50 (= 5,000 × $0.0025) per contract.
- *Daily price limits.* The exchange sets the maximum price movement for a contract during a day. For example, wheat cannot move more than $0.20 from its close the preceding day, for a daily price limit of $1,000. When a contract moves down by its daily price limit, it is said to be **limit down**. When the contract moves up by its price limit, it is said to be **limit up**.
- *Position limits.* The exchange sets a maximum number of contracts that a speculator may hold in order to prevent speculators from having an undue influence on the market. Such limits do not apply to hedgers.

Futures/Spot Convergence

AIM 24.2: Explain the convergence of futures and spot prices.

The spot (cash) price of a commodity or financial asset is the price for immediate delivery. The futures price is the price today for delivery at some future point in time (i.e., the maturity date). The **basis** is the difference between the spot price and the futures price.

basis = spot price – futures price

As the maturity date nears, the basis converges toward zero. At expiration, the spot price must equal the futures price because the futures price has become the price today for delivery today, which is the same as the spot. Arbitrage will force the prices to be the same at contract expiration.

Example: Why the futures price must equal the spot price at expiration

Suppose the current spot price of silver is $4.65. Demonstrate by arbitrage that the futures price of a futures silver contract that expires in one minute must equal the spot price.

Answer:

Suppose the futures price was $4.70. We could buy the silver at the spot price of $4.65, sell the futures contract, and deliver the silver under the contract at $4.70. Our profit would be $4.70 – $4.65 = $0.05. Because the contract matures in one minute, there is virtually no risk to this arbitrage trade.

Suppose instead the futures price was $4.61. Now we would buy the silver contract, take delivery of the silver by paying $4.61, and then sell the silver at the spot price of $4.65. Our profit is $4.65 – $4.61 = $0.04. Once again, this is a riskless arbitrage trade.

Therefore, in order to prevent arbitrage, the futures price at the maturity of the contract must be equal to the spot price of $4.65.

Operation of Margins

AIM 24.3: Describe the rationale for margin requirements and explain how they work.

Margin is cash or highly liquid collateral placed in an account to ensure that any trading losses will be met. Marking to market is the daily procedure of adjusting the margin account balance for daily movements in the futures price. The amount required to open a futures position is called the **initial margin**. The **maintenance margin** is the minimum margin account balance required to retain the futures position. When the margin account balance

falls below the maintenance margin, the investor gets a margin call, and he must bring the margin account back to the initial margin amount. The amount necessary to do this is called the **variation margin**.

Example: Margin trading

Let's return to our investor with the long gold contract. The investor entered the position at $993.60. Each contract controls 100 troy ounces for a current market value of $99,360. Assume that the initial margin is $2,500, the maintenance margin is $2,000, and the futures price drops to $991.00 at the end of the first day and $985.00 on the end of the second day. **Compute** the amount in the margin account at the end of each day for the long position and any variation margin needed.

Answer:

At the end of the first day, the loss is computed as ($991 – $993.6)100 = –$260, so when the account is marked to market, $260 is withdrawn from the buyer's margin account and $260 deposited in the seller's margin account. The buyer's (long) margin account balance is now $2,240 (= $2,500 – $260). The margin account balance for the short position is now $2,760 (= $2,500 + $260).

At the end of the second day, the daily loss is ($985 – $991)100 = –$600, and the buyer's margin account balance is reduced to $1,640 (= $2,240 – $600). At $1,640 the investor will get a margin call since the margin account balance is less than the maintenance margin. The variation margin is the amount necessary to bring the margin account back up to the initial margin. In this case, it is $860 (= $2,500 – $1,640).

Depending on the client, brokers may require the posting of a balance in the margin account more than the maintenance margin requirements established by exchanges. For example, hedgers are usually required to post smaller margins than speculators. To ensure that the daily cash flows are withdrawn or contributed appropriately, the exchange has a clearinghouse.

Clearinghouse

AIM 24.4: Describe the role of a clearinghouse in futures transactions.

Each exchange has a **clearinghouse**. The clearinghouse guarantees that traders in the futures market will honor their obligations. The clearinghouse does this by splitting each trade once it is made and acting as the opposite side of each position. The clearinghouse acts as the buyer to every seller and the seller to every buyer. By doing this, the clearinghouse allows either side of the trade to reverse positions at a future date without having to contact the other side of the initial trade. This allows traders to enter the market knowing that they will be able to reverse their position. Traders are also freed from having to worry about the counterparty defaulting since the counterparty is now the clearinghouse. In the history of U.S. futures trading, the clearinghouse has never defaulted on a trade.

The clearinghouse has members that collateralize it, ensuring that no defaults take place. All trades eventually go through the clearinghouse members, who must have a **clearing margin** posted at the clearinghouse in the same way an investor has a margin account with a broker. This ensures that the clearinghouse is liquid enough at all times to honor all obligations under futures contracts.

Collateralization

AIM 24.5: Describe the role of collateralization in the over-the-counter market and compare it to the margining system.

The over-the-counter (OTC) market includes the trading in all securities not listed on one of the registered exchanges. This market is subject to a good deal of credit risk since the party on the other side of an OTC contract could default on its payments. One way to reduce this credit risk is by means of **collateralization**. Collateralization is basically a marked to market feature for the OTC market where any loss is settled in cash at the end of the trading day. A cash payment is made to the party with a positive account balance. This is a similar system to trading on margin where the futures trader needs to restore funds if the value of the contract drops below the maintenance margin.

Normal and Inverted Futures Market

AIM 24.6: Identify and describe the differences between a normal and inverted futures market.

The **settlement price** is analogous to the closing price for a stock but is not simply the price of the last trade. It is an average of the prices of the trades during the last period of trading, called the closing period, which is set by the exchange. This feature of the settlement price prevents manipulation by traders. The settlement price is used to make margin calculations at the end of each trading day.

Depending on the direction of futures settlement prices, the market may be normal or inverted. Increasing settlement prices over time indicates a **normal market**. Conversely, decreasing settlement prices over time indicates an **inverted market**.

The Delivery Process

AIM 24.7: Describe the mechanics of the delivery process and contrast it with cash settlement.

There are four ways to terminate a futures contract:

1. A short can terminate the contract by delivering the goods, a long by accepting delivery and paying the contract price to the short. This is called **delivery**. The location for delivery (for physical assets), terms of delivery, and details of exactly what is to be delivered are all specified in the **notice of intention to deliver** file. Each exchange has specific rules as to the conditions for making an intent to deliver. However, the price paid or received will be dictated by the settlement period on the exchange-determined last trading day of the contract.

2. In a **cash-settlement contract**, delivery is not an option. The futures account is marked to market based on the settlement price on the last day of trading.

3. You may make a **reverse**, or **offsetting**, trade in the futures market. With futures, the other side of your position is held by the clearinghouse—if you make an exact opposite trade (maturity, quantity, and good) to your current position, the clearinghouse will net your positions out, leaving you with a zero balance. This is how most futures positions are settled. The contract price can differ between the two contracts. If you initially are long one contract at $970 per ounce of gold and subsequently sell (i.e., take the short position in) an identical gold contract when the price is $950 per ounce, $20 multiplied by the number of ounces of gold specified in the contract will be deducted from the margin deposit(s) in your account. The sale of the futures contract ends the exposure to future price fluctuations on the first contract. Your position has been *reversed*, or **closed out**, by a *closing* trade.

4. A position may also be settled through an **exchange for physicals**. Here you find a trader with an opposite position to your own and deliver the goods and settle up between yourselves, off the floor of the exchange (i.e., an ex-pit transaction). This is the sole exception to the federal law that requires that all trades take place on the floor of the exchange. You must then contact the clearinghouse and tell them what happened. An exchange for physicals differs from a delivery in that the traders actually exchange the goods, the contract is not closed on the floor of the exchange, and the two traders privately negotiate the terms of the transaction. Regular delivery involves only one trader and the clearinghouse.

Types of Orders

AIM 24.8: Define and demonstrate an understanding of the impact of different order types, including: market, limit, stop-loss, stop-limit, market-if-touched, discretionary, time-of-day, open, and fill-or-kill.

There are several different types of orders in the marketplace:

Market orders are orders to buy or sell at the best price available. A **discretionary order** is a market order where the broker has the option to delay transaction in search of a better price.

Limit orders are orders to buy or sell away from the current market price. A *limit buy order* is placed below the current price. A *limit sell order* is placed above the current price. Limit orders have a time limit, such as instantaneous, one day, one week, one month, or good till canceled. Limit orders are turned over to the specialist by the commission broker.

Stop-loss orders are used to prevent losses or to protect profits. Suppose you own a stock currently selling for $40. You are afraid that it may drop in price, and if it does, you want your broker to sell it, thereby limiting your losses. You would place a *stop loss sell* order at a specific price (e.g., $35); if the stock price drops to this level, your broker will place a sell market order. A *stop loss buy* order is usually combined with a short sale to limit losses. If the stock price rises to the "stop" price, the broker enters a market order to buy the stock.

Variations on these order types also exist. **Stop-limit orders** are a combination of a stop and limit order. The stop price and limit price must be specified, so that once the stop level is reached, or bettered, the order would turn into a limit order and hopefully transact at the limit price. **Market-if-touched orders**, or MIT orders, are orders that would become market orders once a specified price is reached in the marketplace.

For those orders that remain outstanding until the designated price range is reached, the trader making the order needs to indicate the time period for the order (**time-of-day order**). **Good-till-canceled** (GTC) **orders** (a.k.a. **open orders**) are orders that remain open until they either transact or are canceled. A popular method of submitting a limit order is to have it automatically canceled at the end of the trading day in which it was submitted. **Fill-or-kill orders** must be executed immediately or the trade will not take place.

Regulatory, Accounting, and Tax Frameworks

AIM 24.9: Describe the regulatory, accounting, and tax framework for the U.S.

Regulation

In the United States, the **Commodity Futures Trading Commission** (CFTC) is responsible for regulating futures markets. The CFTC licenses futures exchanges as well as traders who offer futures trading services to the public. It also approves new futures contracts and any revisions to existing futures contracts. When approving contracts, the agency ensures that each contract serves a useful economic purpose (e.g., for either hedging or speculating).

In addition, the CFTC is responsible for communicating prices to the public, addressing public complaints, and taking disciplinary actions against members who violate futures exchange rules.

Other regulatory bodies that influence the futures markets include the National Futures Association (NFA), the Securities and Exchange Commission (SEC), the Federal Reserve Board, and the U.S. Treasury Department. The SEC, Fed, and Treasury Department are mainly concerned with how futures trading impacts spot market trading in stocks and bonds. The NFA has a more prominent role by attempting to prevent fraud and ensuring that futures markets operate in the best interests of the public. Examples of futures trading fraud include cornering the market (i.e., taking excessive long positions while influencing the supply of the commodity underlying the long futures contracts) and front running (traders using privileged information to trade in their own accounts before customer accounts).

Accounting

When accounting for changes in the market value of a futures contract, changes must be recognized when they occur. The exception to this accounting standard is when a futures contract is being used for hedging purposes. **Hedge accounting** specifies that gains/losses from a hedging instrument be recognized in the same period as gains/losses from the asset being hedged.

Under FAS 133 [Financial Accounting Standard Board (FASB) Statement No. 133], the fair market value of all derivative contracts must be included on the balance sheet. In addition to more position transparency, FAS 133 places stricter guidelines on the use of hedge accounting. To use this accounting method, it must be shown that the hedging instrument frequently and effectively offsets the intended risk exposure.

Taxes

Regarding U.S. tax regulations, differences arise due to the nature of taxable gains/losses and the timing of realized gains/losses. For corporate taxpayers, capital gains are taxed at the same level as ordinary income and capital losses are restricted. For non-corporate taxpayers, capital gains are taxed at the same level as ordinary income, but long-term gains (investments held over one year) are subject to a maximum 15% tax rate. Another difference is that capital losses are deductible for non-corporate taxpayers.

For tax purposes, futures contracts are considered closed out at the end of each year. This gives rise to a 60/40 rule for non-corporate taxpayers where capital gains/losses are treated as 60% long term and 40% short term. This rule, however, does not apply to hedging activities. Using futures for hedging purposes must be declared on the same day the transaction is entered. Gains/losses on hedging transactions are taxed at the same rate as ordinary income.

Key Concepts

1. A long (short) futures position obligates the owner to buy (sell) the underlying asset at a specified price and date.

2. Most futures positions are reversed (or closed out) as opposed to satisfying the contract by making (or taking) delivery.

3. Futures are traded on margin (leveraged):
   - Initial margin is the necessary collateral to trade the futures.
   - Maintenance margin is the minimum collateral amount required to retain trading privileges.
   - Variation margin is the collateral amount that must be deposited to replenish the margin account back to the initial margin.

4. The futures market is a zero-sum game in that the short's losses are the long's gains and vice versa. Gains and losses due to changes in futures prices are computed at the end of each trading day in a process known as marking to market.

5. The clearinghouse maintains an orderly and liquid market by acting as the counterparty to each long or short futures position.

6. Several different types of orders exist in the marketplace including: market, limit, stop-loss, stop-limit, and market-if-touched orders.

7. The Commodity Futures Trading Commission (CFTC) is responsible for regulating futures markets.

8. When accounting for changes in the market value of a futures contract, changes must be recognized when they occur.

9. For corporate taxpayers, capital gains on futures are taxed at the same level as ordinary income. For non-corporate taxpayers, capital gains are taxed at the same level as ordinary income, but long-term gains are subject to a maximum 15% tax rate.

Concept Checkers

1. When an investor is obligated to buy the underlying asset in a futures position, it is a:
   A. basis trade.
   B. long-futures position.
   C. short-futures position.
   D. hedged-futures position.

2. Which of the following are characteristics specified by a futures contract?
   I. Asset quality and asset quantity.
   II. Delivery arrangements and delivery time.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

3. An investor enters into a short position in a gold futures contract with the following characteristics:
   - The initial margin is $3,000.
   - The maintenance margin is $2,250.
   - The contract price is $1,300.
   - Each contract controls 100 troy ounces.

   If the price drops to $1,295 at the end of the first day and $1,290 at the end of the second day, which of the following is closest to the variation margin required at the end of the second day?
   A. $0.
   B. $250.
   C. $500.
   D. $1,000.

4. Which of the following items are functions of the clearinghouse?
   I. Determine which contracts trade.
   II. Receive margin deposits from brokers.
   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

5. It is possible that which of the following types of orders may never be executed?
   A. Limit orders.
   B. Market-if-touched (MIT) orders.
   C. Stop-limit orders.
   D. All of the above.

Concept Checker Answers

1. **B** When an investor is obligated to buy the underlying asset in a futures position, it is a long futures position.

2. **C** Delivery time, asset quality, asset quantity, and delivery arrangements are all characteristics specified by the futures contract.

3. **A** Note that the investor in this question has a short position that profits from price declines. The short position margin account has increased by $1,000 over the two days, so there is no variation margin required.

4. **B** The clearinghouse acts as buyer to every seller and seller to every buyer, thus virtually eliminating default risk. It also collects margin payments from clearing members (brokers). Determining which contracts will trade is a function of the exchange, not the clearinghouse.

5. **D** All of these orders require that the price reach a certain range before being activated. If the price never reaches that range, the order will never be activated.

The following is a review of the Financial Markets and Products principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Hedging Strategies Using Futures

Topic 25

Exam Focus

Futures contracts are used extensively for implementing hedging strategies. This topic presents the calculations for determining the optimal hedge ratio and shows how to use it to determine the number of futures contracts necessary to hedge a spot market exposure. This topic also addresses basis risk, the change in the relationship between spot prices and futures prices over a hedge horizon. Basis risk arises because an asset being hedged may not be exactly the same as the asset underlying the futures contract.

Hedging With Futures

AIM 25.1: Define and differentiate between short and long hedges and identify appropriate uses.

A **short hedge** occurs when the hedger shorts (sells) a futures contract to hedge against a price decrease in the existing long position. When the price of the hedged asset decreases, the short futures position realizes a positive return, offsetting the decline in asset value. Therefore, a short hedge is appropriate when you have a long position and expect prices to decline.

A **long hedge** occurs when the hedger buys a futures contract to hedge against an increase in the value of the asset that underlies a short position. In this case, an increase in the value of the shorted asset will result in a loss to the short seller. The objective of the long hedge is to offset the loss in the short position with a gain from the long futures position. A long hedge is therefore appropriate when you have a short position and expect prices to rise.

Advantages and Disadvantages of Hedging

AIM 25.2: Describe the arguments for and against hedging and the potential impact of hedging on firm profitability.

The objective of hedging with futures contracts is to reduce or eliminate the price risk of an asset or a portfolio. For example, a farmer with a large corn crop that will be harvested in a few months could wait until the end of the growing season and sell his corn at the prevailing spot price, *or* he could sell corn futures and "lock in" the price of his corn at a predetermined rate. By taking a short position in a corn futures contract, the farmer eliminates—or at least reduces—exposure to fluctuating corn prices. This is an example of a *short hedge*, where the user locks in a future selling price.

Alternatively, a cereal company will need to purchase corn in the future. The company could wait to buy corn in the spot market and face the volatility of future corn spot prices or lock in its purchase price by buying corn futures in advance. This demonstrates an *anticipatory hedge*. The cereal company has an anticipated need for corn and buys corn futures to lock in the price of those future corn purchases. This is an example of a *long hedge*, where the user locks in a future purchasing price.

It is easy to see that the benefit from hedging leads to less uncertainty regarding future profitability. However, there are some arguments against hedging. The main issue is that hedging can lead to less profitability if the asset being hedged ends up increasing in value. The increase in value will be offset by a corresponding loss in the futures contract used for the hedge.

Another argument against hedging is the questionable benefit that accrues to shareholders. Clearly, hedging reduces risk for a company and its shareholders, but there is reason to believe that shareholders can more easily hedge risk on their own. A third argument deals with the nature of the hedging company's industry. For example, assume that prices in an industry frequently adjust for changes in input prices and exchange rates. If competitors do not hedge, then there is an incentive to keep the status quo. In this way, the company ensures that profitability will remain more stable than if it were to hedge frequent changes.

Basis Risk

AIM 25.3: Define the basis and the various sources of basis risk, and explain how basis risks arise when hedging with futures.

AIM 25.4: Define cross hedging, and compute and interpret the minimum variance hedge ratio and hedge effectiveness.

When all of the existing position characteristics match perfectly with those of the futures contract specifications, we have a perfect hedge. With a perfect hedge, the loss on a hedged position will be perfectly offset by the gain on the futures position. Perfect hedges are not very common. There are two major reasons why this is so: (1) the asset in the existing position is often not the same as that underlying the futures (e.g., we may be hedging a corporate bond portfolio with a futures contract on a U.S. Treasury bond), and (2) the hedging horizon may not match perfectly with the maturity of the futures contract. The existence of either one of these conditions leads to what is called **basis risk**.

The basis in a hedge is defined as the difference between the spot price on a hedged asset and the futures price of the hedging instrument (e.g., futures contract). Basis is calculated as:

basis = spot price of asset being hedged – futures price of contract used in hedge

When the hedged asset and the asset underlying the hedging instrument are the same, the basis will be zero at maturity.

Professor's Note: This is the typical definition for basis (where basis equals spot price minus futures price). However, basis is also sometimes defined as: futures price minus spot price, mostly when dealing with financial asset futures.

When the spot price increases faster than the futures price over the hedging horizon, basis increases and a strengthening of the basis is said to occur. When the futures price increases faster than the spot price and the basis decreases, a weakening of the basis occurs. When hedging, a change in basis is unavoidable. The change in basis over the hedge horizon is termed *basis risk*, and it can work either for or against a hedger.

To minimize basis risk, hedgers should select the contract on an asset that is most highly correlated with the spot position and a contract maturity that is closest to the hedging horizon. Contract liquidity must also be considered when selecting a futures contract for hedging.

Three sources of basis risk are: (1) interruption in the convergence of the futures and spot prices, (2) changes in the cost of carry, and (3) imperfect matching between the cash asset and the hedge asset. Let's discuss each of these sources in more detail.

1. *Interruption in the convergence of the futures and spot prices.* Normally, spot prices and futures prices will converge as the time to maturity decreases, and basis reduces to zero at maturity. However, if the position is unwound prior to maturity, the return to the futures position could be different from the return to the cash position. A more rapid convergence results in a more rapid transfer of margin payments, while a less rapid convergence would delay payments. An interruption in the convergence could result in payments from the seller to the buyer. All of these effects are types of basis risk.

2. *Changes in the cost of carry.* Significant basis risk can arise due to changes in the components of the cost of carry. The cost of carry includes storage and safekeeping, interest, insurance, and related costs. Perhaps the most volatile of these costs is interest costs. An increase in the interest rates increases the opportunity cost of holding the asset, so the cost of carry and, hence, the basis of the contract rises.

3. *Imperfect matching between the cash asset and the hedge asset.* Sometimes it may be more efficient to **cross hedge** or hedge a cash position with a hedge asset that is closely related but different from the cash asset. For example, Eurodollar deposits are closely related to T-bill rates and may be considered a good hedge. However, if there is a structural shock that changes the close relationship of these two assets, the position may not be hedged as effectively as originally believed. This is the most common form of basis risk. Other forms of mismatch include maturity or duration mismatches, liquidity mismatches, and credit risk mismatches:
   - *Maturity or duration mismatch.* Hedging a portfolio of mortgages with 10-year Treasury notes (T-notes) may seem reasonable if the effective duration of the mortgages matches the duration of the T-notes. However, if rates fall and the mortgages prepay faster (resulting in a shorter duration), the position will not be matched.
   - *Liquidity mismatch.* Hedging an illiquid asset with a more liquid one will result in greater basis risk. Although over the long term the prices may be comparable, the difference in liquidity may result in large gaps between the pricing of the two assets. Hence, basis risk is inversely proportional to the liquidity of the hedged asset.

- *Credit risk mismatch.* The widening or narrowing of credit spreads constitutes another form of basis risk when the credit risk of the hedged asset is different (or becomes different) from the credit risk of the hedge instrument.

All of these represent basis risk. The size and type of basis risk can vary during the term of the contract, even if the position is perfectly hedged at maturity.

The Optimal Hedge Ratio

We can account for an imperfect relationship between the spot and futures positions by calculating an **optimal hedge ratio** that incorporates the degree of correlation between the rates.

A hedge ratio is the ratio of the size of the futures position relative to the spot position. The *optimal hedge ratio*, which minimizes the variance of the combined hedge position, is defined as follows:

$$\text{HR} = \rho_{S,F}\frac{\sigma_S}{\sigma_F}$$

This is also the beta of spot prices with respect to futures contract prices since:

$$\rho = \frac{\text{Cov}_{S,F}}{\sigma_S\sigma_F} \text{ and } \frac{\text{Cov}_{S,F}}{\sigma_S\sigma_F}\times\frac{\sigma_S}{\sigma_F} = \frac{\text{Cov}_{S,F}}{\sigma_F^{\ 2}} = \beta_{S,F}$$

where:
$\rho_{S,F}$ = the correlation between the spot prices and the futures prices
σ_S = the standard deviation of the spot price
σ_F = the standard deviation of the futures price

Example: Minimum variance hedge ratio

Suppose a currency trader computed the correlation between the spot and futures to be 0.925, the annual standard deviation of the spot price to be \$0.10, and the annual standard deviation of the futures price to be \$0.125. Compute the hedge ratio.

Answer:

$$\text{HR} = 0.925\times\frac{0.100}{0.125} = 0.74$$

The ratio of the size of the futures to the spot should be 0.74.

The **effectiveness of the hedge** measures the variance that is reduced by implementing the optimal hedge. This effectiveness can be evaluated with a coefficient of determination (R^2) term where the independent variable is the change in futures prices and the dependent variable is the change in spot prices. Recall that R^2 measures the goodness-of-fit of a regression. As shown previously, the beta of spot prices with respect to futures prices is equal

to the hedge ratio (HR), which is also the slope of this regression. The R^2 measure for this simple linear regression is the square of the correlation coefficient (ρ^2) between spot and futures prices.

Hedging With Stock Index Futures

AIM 25.5: Define, compute and interpret the optimal number of futures contracts needed to hedge an exposure, including a "tailing the hedge" adjustment.

A common hedging application is the hedging of equity portfolios using futures contracts on stock indices (index futures). In this application, it is important to remember that the hedged portfolio's beta serves as a hedge ratio when determining the correct number of contracts to purchase or sell. The number of futures contracts required to completely hedge an equity position is determined with the following formula:

$$\text{number of contracts} = \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{value of futures contract}}\right)$$

$$= \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}}\right)$$

Example: Hedging with stock index futures

You are a portfolio manager with a $20 million growth portfolio that has a beta of 1.4, relative to the S&P 500. The S&P 500 futures are trading at 1,150, and the multiplier is 250. You would like to hedge your exposure to market risk over the next few months. **Identify** whether a long or short hedge is appropriate, and **determine** the number of S&P 500 contracts you need to implement the hedge.

Answer:

You are long the S&P 500, so you should construct a short hedge and sell the futures contract. The number of contracts to sell is equal to:

$$1.4 \times \frac{\$20{,}000{,}000}{1{,}150 \times 250} \approx 97 \text{ contracts}$$

Tailing the Hedge

A hedger may actually over-hedge the underlying exposure if daily settlement is not properly accounted for. To correct for the possibility of over-hedging, a hedger can implement a **tailing the hedge** strategy. The extra step needed to carry out this strategy is to multiply the hedge ratio by the daily spot price to futures price ratio. In practice, it is not efficient to adjust the hedge for every daily change in the spot-to-futures ratio.

Example: Tailing the hedge

Suppose that you would like to make a tailing the hedge adjustment to the number of contracts needed in the previous example. Assume that when evaluating the next daily settlement period you find that the S&P 500 spot price is 1,095 and the futures price is now 1,160. Determine the number of S&P 500 contracts needed after making a tailing the hedge adjustment.

Answer:

The number of contracts to sell is equal to:

$$1.4 \times [(\$20{,}000{,}000) / (1{,}150 \times 250)] \times (1{,}095 / 1{,}160) = 92 \text{ contracts}$$

Adjusting the Portfolio Beta

AIM 25.6: Demonstrate how to use stock index futures contracts to change a stock portfolio's beta.

Hedging an existing equity portfolio with index futures is an attempt to reduce the *systematic risk* of the portfolio. If the beta of the capital asset pricing model is used as the systematic risk measure, then hedging boils down to a reduction of the portfolio beta. Let β be our portfolio beta, β^* be our target beta after we implement the strategy with index futures, P be our portfolio value, and A be the value of the underlying asset. To compute the appropriate number of futures, we use the following equation:

$$\text{number of contracts} = (\beta^* - \beta)\frac{P}{A}$$

This equation can result in either positive or negative values. Negative values indicate selling futures (decreasing systematic risk), and positive values indicate buying futures contracts (increasing systematic risk).

Example: Adjusting portfolio beta

Suppose we have a well-diversified $100 million equity portfolio. The portfolio beta relative to the S&P 500 is 1.2. The current value of the S&P 500 Index is 1,080. The portfolio manager wants to completely hedge the systematic risk of the portfolio over the next three months using S&P 500 Index futures. Demonstrate how to adjust the portfolio's beta.

Answer:

In this instance, our target beta, β*, is 0, since a complete hedge is desired.

$$\text{number of contracts} = (0 - 1.2)\frac{100{,}000{,}000}{1{,}080 \times 250} = -444.44$$

The negative sign tells us we need to sell 444 contracts.

Rolling a Hedge Forward

AIM 25.7: Describe what is meant by "rolling the hedge forward" and discuss some of the risks that arise from such a strategy.

When the hedging horizon is long relative to the maturity of the futures used in the hedging strategy, hedges have to be rolled forward as the futures contracts in the hedge come to maturity or expiration. Typically, as a maturity date approaches, the hedger must close out the existing position and replace it with another contract with a later maturity. This is called **rolling the hedge forward**.

When rolling a hedge forward, hedgers are not only exposed to the basis risk of the original hedge, they are also exposed to the basis risk of a new position each time the hedge is rolled forward. This is referred to as rollover basis risk, or simply **rollover risk**.

Key Concepts

1. Hedging may be achieved by shorting futures to protect an underlying position against price deterioration or by buying futures to hedge against unanticipated price increases in an underlying asset.

2. Basis risk is the risk that a difference may occur between the spot price of a hedged asset and the futures price of the contract used to implement the hedge.

3. Basis risk is zero only when there is a perfect match between the hedged asset and the contract's underlying instrument in terms of maturity and asset type.

4. A hedge ratio is the ratio of the size of the futures position relative to the spot position necessary to provide a desired level of protection.

5. When hedging an equity portfolio with a short position in stock index futures, the beta of the portfolio is reduced.

6. When the hedging horizon is longer than the maturity of the futures, the hedge must be rolled forward to retain the hedge. This exposes the hedger to rollover risk, the basis risk when the hedge is re-established.

Concept Checkers

Use the following data to answer Questions 1 and 2.

An equity portfolio is worth $100 million with the benchmark of the Dow Jones Industrial Average. The Dow is currently at 10,000, and the corresponding portfolio beta is 1.2. The futures multiplier for the Dow is 10.

1. Which of the following is the closest to the number of contracts needed to double the portfolio beta?
   A. 1,100.
   B. 1,168.
   C. 1,188.
   D. 1,200.

2. To cut the beta in half, the correct trade is:
   A. long 600 contracts.
   B. short 600 contracts.
   C. long 1,200 contracts.
   D. short 1,200 contracts.

3. Which of the following situations describe a hedger with exposure to basis risk?
   I. A portfolio manager for a large-cap growth fund knows he will be receiving a significant cash investment from a client within the next month and wants to pre-invest the cash using stock index futures.
   II. A farmer has a large crop of corn he is looking to sell before June 30. The farmer uses a June futures contract to lock in his sales price.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

4. The standard deviation of price changes in a wheat futures contract is 0.6, while the standard deviation of changes in the price of wheat is 0.75. The covariance between the spot price changes and the futures price changes is 0.3825. Which of the following is closest to the optimal hedge ratio?
   A. 0.478.
   B. 0.850.
   C. 1.063.
   D. 1.250.

5. A large-cap value equity manager has a $6,500,000 equity portfolio with a beta of 0.92. An S&P 500 futures contract is available with a current value of 1,175 and a multiplier of 250. What position should the manager take to completely hedge the portfolio's market risk?
   A. Short 20 contracts.
   B. Short 22 contracts.
   C. Short 24 contracts.
   D. Long 22 contracts.

CONCEPT CHECKER ANSWERS

1. D $(2.4-1.2)\dfrac{100{,}000{,}000}{10{,}000\times 10}=1.2\times 1{,}000=1{,}200$

   where beta = 1.2, target beta = 2.4, A = 10 × 10,000, P = $100 million

2. B $(0.6-1.2)\dfrac{100{,}000{,}000}{10{,}000\times 10}=-0.6\times 1{,}000=-600$

   where beta = 1.2, target beta = 0.6, A = 10 × 10,000, P = $100 million

3. C Both of these situations describe exposure to basis risk—the risk that the difference between the spot price and futures delivery price will change. The portfolio manager using futures to pre-invest the cash does not know the exact date he will receive the cash and may need to sell or hold the futures contract for a longer time period than intended. The farmer may need to sell his June futures contract early if he sells his corn earlier than the June futures expiration date.

4. C Notice in this problem, we were given the covariance but not the correlation. We can calculate the correlation using the formula learned back in the Quantitative Analysis material, as follows:

   $$\rho=\frac{COV_{S,F}}{(\sigma_S)(\sigma_F)}=\frac{0.3825}{(0.75)(0.60)}=0.85$$

   Now that we have our correlation value, we can calculate the minimum hedge ratio as:

   $$0.85\left(\frac{0.75}{0.60}\right)=1.0625,\ \text{or, directly,}\ \frac{Cov_{S,F}}{{\sigma_F}^2}=\frac{0.3825}{0.6^2}=1.0625$$

5. A $0.92\times\dfrac{6{,}500{,}000}{1{,}175\times 250}\approx 20$ contracts

   Because the manager has a long position in the market, she will want to take a short position in the futures.

The following is a review of the Financial Markets and Products principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Interest Rates

Topic 26

Exam Focus

Spot, or zero, rates are computed from coupon bonds using a method known as bootstrapping. Forward rates can then be computed from the spot or zero curve. For the exam, understand how to use the bootstrapping method and how to compute forward rates from spot rates. Also, be familiar with the discrete and continuous compounding methods. Note that the fixed income readings in Book 4 will provide more information on the calculation of spot and forward rates as well as constructing the spot and forward rate curves. Duration and convexity are also mentioned in this topic but will be discussed in much more detail in Book 4.

Types of Rates

AIM 26.2: Describe Treasury Rates, LIBOR, Repo Rates, and what is meant by the risk-free rate.

Three interest rates play a key role in interest rate derivatives: Treasury rates, LIBOR, and repo rates. Keep in mind that interest rates increase as the credit risk of the underlying instrument increases.

- **Treasury rates.** Treasury rates are the rates that correspond to government borrowing in its own currency. They are considered risk-free rates.
- **LIBOR.** LIBOR is the rate at which large international banks fund their activities. Some credit risk exists with LIBOR.
- **Repo rates.** The "repo" or repurchase agreement rate is the implied rate on a repurchase agreement. In a repo agreement, one party agrees to sell a security to another with the understanding that the selling party will buy it back later at a specified higher price. The interest rate implied by the price differential is the repo rate. The most common repo is the overnight repurchase agreement. Longer-term agreements are called term repos. Depending on the parties and structure involved, there is some credit risk with repurchase agreements.

Professor's Note: You may see reference to an inverse floater (a.k.a. reverse floater) on the exam. Just know that an inverse floater is a debt instrument whose coupon payments fluctuate inversely with the reference rate (e.g., LIBOR). For example, the inverse floater's coupon rate will increase when LIBOR decreases and vice versa.

As mentioned, Treasury rates (such as T-bill and T-bond rates) are often considered the benchmark for nominal risk-free rates. However, derivative traders view these rates as being too low to be considered risk free (since part of the demand for these bonds comes from fulfilling regulatory requirements, which drives prices up and rates down). As a result, traders instead use LIBOR rates for short-term risk-free rates, because LIBOR better reflects a trader's opportunity cost of capital.

Compounding

AIM 26.1: Calculate the value of an investment using daily, weekly, monthly, quarterly, semiannual, annual, and continuous compounding. Convert rates based on different compounding frequencies.

Derivative pricing often uses a framework called continuous time mathematics. In this framework, it is assumed that returns are continuously compounded. This is a theoretical construct only, as returns cannot literally be compounded continuously. Fortunately, converting discrete compounding to continuous compounding is straightforward.

If we have an initial investment of *A* that earns an annual rate *R*, compounded *m* times a year for *n* years, then it has a future value of:

$$FV_1 = A\left(1+\frac{R}{m}\right)^{m\times n}$$

If our same investment is continuously compounded over that period, it has a future value of:

$$FV_2 = Ae^{R\times n}$$

For any rate, *R*, FV_2 will always be greater than FV_1. The difference will decrease as *m* increases. In fact, as *m* becomes infinitely large, the difference goes to zero.

In most circumstances rates are discretely compounded, so we need to find the continuously compounded rate that gives the same future value. Using the previous two equations, the goal is to solve the following:

$$A\left(1+\frac{R}{m}\right)^{m\times n} = Ae^{R_c n}$$

where:
R_c = the continuous rate

We can solve for R_c as:

$$R_c = m\times \ln\left(1+\frac{R}{m}\right)$$

We can also solve for *R* as:

$$R = m\left(e^{R_c/m}-1\right)$$

Professor's Note: In order to algebraically solve for R or R_c, given one of the equations above, it is helpful to understand that e is the base of the natural log (ln). In other words, the natural log is the inverse function of the exponential function: $e^{ln(x)} = ln(e^x) = x$. So if you are given an equation such that $R = e^x$; x will be equal to: ln(R).

Example: Computing continuous rates

Suppose we have a 5% rate that is compounded semiannually. Compute the corresponding continuous rate. Repeat this for quarterly, monthly, weekly, and daily compounding.

Answer:

$$R_c = 2\ln\left(1+\frac{0.05}{2}\right) = 0.049385$$

The results for other compounding frequencies are shown in Figure 1.

Figure 1: Compounding Frequencies and Returns

m	R_c
4	0.049690
12	0.049896
52	0.049976
250	0.049995

Notice that as m increases, the difference between the rates decreases.

Example: Discrete compounding rate

A loan is quoted at 12% annually with continuous compounding. Interest is paid monthly. Calculate the equivalent rate with monthly compounding.

Answer:

$$R = 12(e^{0.12/12} - 1) = 12.06\%$$

Spot (Zero) Rates and Bond Pricing

AIM 26.3: Calculate the theoretical price of a coupon paying bond using spot rates.

Spot rates are the rates that correspond to zero-coupon bond yields. They are the appropriate discount rates for a single cash flow at a particular future time or maturity. Spot rates are also often called zero rates. Most interest rates that are observed in the market, such as coupon bond yields, are not spot rates.

Bond Pricing

A coupon bond makes a series of cash flows. Each cash flow considered in isolation is equivalent to a zero-coupon bond. Using this interpretation, a coupon bond is a series of zero-coupon bonds, and its value, assuming continuous compounding and semiannual coupons, is:

$$B = \left(\frac{c}{2} \times \sum_{j=1}^{N} e^{-\frac{z_j}{2} \times j} \right) + \left(FV \times e^{-\frac{z_N}{2} \times N} \right)$$

where:
c = the annual coupon
N = the number of semiannual payment periods
z_j = the bond equivalent spot rate that corresponds to j periods (j/2 years) on a continuously compounded basis
FV = the face value of the bond

Don't let this formula intimidate you. It simply says that the value of a bond is the present value of its cash flows, where each cash flow is discounted at the appropriate spot rate for its maturity. Notice that the negative sign on the rate just means that the coupon and principal payments are being discounted back to the present in a continuous fashion. The following example is a good illustration of the process.

Example: Calculating bond price

Compute the price of a $100 face value, 2-year, 4% semiannual coupon bond using the annualized spot rates in Figure 2.

Figure 2: Spot Rates

Maturity (Years)	*Spot Rate (%)*
0.5	2.5
1.0	2.6
1.5	2.7
2.0	2.9

Answer:

$$B = \left(\$2 \times e^{-\frac{0.025}{2} \times 1} \right) + \left(\$2 \times e^{-\frac{0.026}{2} \times 2} \right) + \left(\$2 \times e^{-\frac{0.027}{2} \times 3} \right) + \left(\$102 \times e^{-\frac{0.029}{2} \times 4} \right) = \$102.10$$

Bond Yield

The yield of a bond is the single discount rate that equates the present value of a bond to its market price. You can use a financial calculator to compute bond yield, as in the following example.

Example: Calculating bond yield

Compute the yield for the bond in the previous example.

Answer:

PMT = 2; N = 4; PV = −102.10; FV = 100; CPT → I/Y = 1.456;
Y = 1.456% × 2 ≈ 2.91%

The bond's **par yield** is the rate which makes the price of a bond equal to its par value. When the bond is trading at par, the coupon will be equal to the bond's yield.

Bootstrapping Spot Rates

The theoretical spot curve is derived by interpreting each Treasury bond (T-bond) as a package of zero-coupon bonds. Using the prices for each bond, the spot curve is computed using the bootstrapping methodology.

For example, suppose there is a T-bond maturing on a coupon date in exactly six months. Further assume that the bond is priced at 102.2969% of par and has a semiannual coupon of 6.125% How is the corresponding spot rate computed? In this case, this is truly a zero-coupon bond, since there is only one cash flow, which occurs in six months. Simply solve for z_1 in the bond valuation equation, given the price, as follows:

$$\$102.2969 = \left(\$100 + \frac{\$6.125}{2}\right) \times e^{-\frac{z_1}{2}}$$

Solving this for z_1:

$$z_1 = -2 \times \ln\left[\frac{\$102.2969}{\left(\$100 + \frac{\$6.125}{2}\right)}\right] = 1.491\%$$

The 6-month spot rate on a bond equivalent basis is 1.491%. Also note that the yield to maturity did not need to be computed in this case because the yield to maturity (YTM) and the spot rate are the same.

How is the spot rate that corresponds to one year found? Suppose a T-bond that matures in one year is priced at 104.0469% of par and has a semiannual coupon of 6.25%. From the previous computation, the 6-month spot rate is known, so the bond valuation equation can be written as:

$$\$104.0469 = \left(\frac{\$6.25}{2} \times e^{-\frac{0.01491}{2}}\right) + \left(\$100 + \frac{\$6.25}{2}\right) \times e^{-\frac{z_2}{2} \times 2}$$

$$\Rightarrow z_2 = 0.02136 = 2.136\%$$

The 1-year spot rate with continuous compounding is 2.136%.

Example: Bootstrapping spot rates

Compute the corresponding spot rate curve using the information in Figure 3. Note that we've already computed the first two spot rates.

Figure 3: Input Information to Bootstrap Spot Rates

Price as a Percentage of Par	*Coupon*	*Semiannual Period*	*Maturity (Years)*
102.2969	6.125	1	0.5
104.0469	6.250	2	1.0
104.0000	5.250	3	1.5
103.5469	4.750	4	2.0

Answer:

The spot rates derived by bootstrapping are shown in Figure 4.

Figure 4: Bootstrapped Spot Rate Curve

Price as a Percentage of Par	*Coupon*	*Semiannual Period*	*Maturity (Years)*	*Spot Rates*
102.2969	6.125	1	0.5	1.491%
104.0469	6.250	2	1.0	2.136%
104.0000	5.250	3	1.5	2.515%
103.5469	4.750	4	2.0	2.915%

An alternative verification is to use the spot rates to check if they result in the same prices using the bond valuation equation. For example, using the spot rates will ensure computation of the same price for the 2-year bond:

$$B = \left(\frac{\$4.75}{2} \times e^{-\frac{0.01491}{2}\times 1}\right) + \left(\frac{\$4.75}{2} \times e^{-\frac{0.02136}{2}\times 2}\right) + \left(\frac{\$4.75}{2} \times e^{-\frac{0.02515}{2}\times 3}\right) +$$

$$\left(\$100 + \frac{\$4.75}{2}\right) \times e^{-\frac{0.02915}{2}\times 4} = \$103.5469$$

This results in a bond price of $103.5469. Notice that this is exactly the price of the 2-year bond.

Forward Rates

AIM 26.4: Calculate forward interest rates from a set of spot rates.

Forward rates are interest rates implied by the spot curve for a specified future period. The spot rates in Figure 4 are the appropriate rates that an investor should expect to realize for various maturities. Suppose an investor is faced with the following two investments, which are based on the spot curve in Figure 4.

1. Invest for two years at 2.915%.
2. Invest for a year at 2.136% and then roll over that investment for another year at the forward rate.

It does not matter which investment is chosen if they both offer the same return at the end of two years. This is the same as stating that both strategies give the same future value at the end of two years. Equating the two future values:

$$e^{\frac{0.02915}{2}\times 4} = e^{\frac{0.02136}{2}\times 2} \times e^{\frac{R_{Forward}}{2}\times 2}$$

where:
$R_{Forward}$ = the 1-year forward rate one year from now

As we will show, for the two strategies to be equal, $R_{Forward}$ must be 3.693%.

We can simplify this calculation by using the following equation:

$$R_{Forward} = \frac{R_2T_2 - R_1T_1}{T_2 - T_1} = R_2 + (R_2 - R_1)\times\left(\frac{T_1}{T_2 - T_1}\right)$$

where:
R_i = the spot rate corresponding with T_i periods
$R_{Forward}$ = the forward rate between T_1 and T_2

For example, if the 1-year rate is 2.136% and the 2-year rate is 2.915%, the 1-year forward rate one year from now is:

$$R_{Forward} = 0.02915 + (0.02915 - 0.02136\ )\times\left(\frac{1}{2-1}\right) = 0.03694 = 3.694\%$$

This is the same forward rate (with slight rounding error) that was calculated before.

As a further example, consider the problem of finding the 1-year forward rate three years from now, given a 3-year spot rate of 7.424% and a 4-year spot rate of 8.216% (both continuously compounded annual rates). Based on the previous formula, the continuously compounded 1-year rate three years from now is:

$$0.08216 + (0.08216 - 0.07424)\times\frac{3}{4-3} = 0.10592$$

With this equation, generalizations can be made between the shape of the spot curve and the forward curve. The second term is always positive for an upward-sloping spot curve. Therefore, when there is an upward-sloping spot curve, the corresponding forward rate curve is upward-sloping and above the spot curve. Similarly, when there is a downward-sloping spot curve, the corresponding forward-rate curve is downward-sloping and below the spot curve.

Forward Rate Agreements

AIM 26.5: Value and calculate the cash flows from a forward rate agreement (FRA).

A **forward rate agreement** (FRA) is a forward contract obligating two parties to agree that a certain interest rate will apply to a principal amount during a specified future time. Obviously, forward rates play a crucial role in the valuation of FRAs. The T_2 cash flow of an FRA that promises the receipt or payment of R_K is:

$$\text{cash flow (if receiving } R_K) = L \times (R_K - R) \times (T_2 - T_1)$$
$$\text{cash flow (if paying } R_K) = L \times (R - R_K) \times (T_2 - T_1)$$

where:

L = principal

R_K = annualized rate on L, expressed with compounding period $T_1 - T_2$

R = annualized actual rate, expressed with compounding period $T_1 - T_2$

T_i = time i, expressed in years

The value of an FRA if we're receiving or paying is:

$$\text{value (if receiving } R_K) = L \times (R_K - R_{Forward}) \times (T_2 - T_1) \times e^{-R_2 \times T_2}$$
$$\text{value (if paying } R_K) = L \times (R_{Forward} - R_K) \times (T_2 - T_1) \times e^{-R_2 \times T_2}$$

where:

$R_{Forward}$ = forward rate between T_1 and T_2

Note that R_2 is expressed as a continuously compounded rate.

Example: Computing the payoff from an FRA

Suppose an investor has entered into an FRA where he has contracted to pay a fixed rate of 3% on $1 million based on the quarterly rate in three months. Assume that rates are compounded quarterly. **Compute** the payoff of the FRA if the quarterly rate is 1% in three months.

Answer:

For this FRA, the payoff will take place in six months. The net payoff will be the difference between the fixed-rate payment and the floating rate receipt. If the floating rate is 1% in three months, the payoff at the end of the sixth month will be:

$$\$1{,}000{,}000\ (0.01 - 0.03)(0.25) = -\$5{,}000$$

Example: Computing the value of an FRA

Suppose the 3-month and 6-month LIBOR spot rates are 4% and 5%, respectively (continuously compounded rates). An investor enters into an FRA in which she will receive 8% (assuming quarterly compounding) on a principal of $5,000,000 between months 3 and 6. **Calculate** the value of the FRA.

Answer:

$$R_{\text{Forward}} = 0.05 + (0.05 - 0.04) \times \left(\frac{1}{2-1}\right) = 0.06 = 6\%$$

$$R_{\text{Forward}}\,(\text{with quarterly compounding}) = 4 \times \left(e^{\frac{0.06}{4}} - 1\right) = 0.060452 = 6.05\%$$

$$\text{value} = \$5{,}000{,}000 \times (0.0800 - 0.0605) \times (0.50 - 0.25) \times e^{-(0.05)(0.5)} = \$23{,}773$$

Duration

AIM 26.6: Describe the limitations of duration and how convexity addresses some of them.

AIM 26.7: Calculate the change in a bond's price given duration, convexity, and a change in interest rates.

The duration of a bond is the average time until the cash flows on the bond are received. For a zero-coupon bond, this is simply the time to maturity. For a coupon bond, its duration will be necessarily shorter than its maturity. The weights on the time in years until each cash flow is to be received are the proportion of the bond's value represented by

each of the coupon payments and the maturity payment. The formula for duration using continuously compounded discounting of the cash flows is:

$$\text{duration} = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-yt_i}}{B}\right]$$

where:
t_i = the time (in years) until cash flow c_i is to be received
y = the continuously compounded yield (discount rate) based on a bond price of B

The usefulness of the duration measure lies in the fact that the approximate change in a bond's price, *B*, for a parallel shift in the yield curve of Δy is:

$$\frac{\Delta B}{B} = -\text{duration} \times \Delta y$$

The change in yield is often expressed as a **basis point** change. One basis point is equivalent to 0.01%. So a 100 basis point change is a change of 1% in the yield.

Modified duration is used when the yield given is something other than a continuously compounded rate. When the yield is expressed as a semiannually compounded rate, for example, modified duration = duration/(1 + y/2). In general we can express this relation as: $\text{modified duration} = \frac{\text{duration}}{1+y/m}$, where *m* is the number of compounding periods per year.

Note that as *m* goes to infinity (continuous compounding), the two measures are equal and there is no difference between the two.

On the exam, you may also see a reference to **dollar duration**. Dollar duration is simply modified duration multiplied by the price of the bond.

Convexity

Duration is a good approximation of price changes for an option-free bond, but it's only good for relatively small changes in interest rates. As rate changes grow larger, the curvature of the bond price/yield relationship becomes more important, meaning that a linear estimate of price changes, such as duration, will contain errors.

In fact, the relationship between bond price and yield is not linear (as assumed by duration) but convex. This convexity shows that the difference between actual and estimated prices widens as the yield swings grow. That is, the widening error in the estimated price is due to the curvature of the actual price path. This is known as the **degree of convexity**.

Fortunately, the amount of convexity in a bond can be measured and used to supplement duration in order to achieve a more accurate estimate of the change in price. It's important to note that all convexity does is account for the amount of error in the estimated price change based on duration. In other words, it picks up where duration leaves off and converts the straight (estimated price) line into a curved line that more closely resembles the convex (actual price) line.

Using Convexity to Improve Price Change Estimates

In order to obtain an estimate of the percentage change in price due to convexity, or the amount of price change that is not explained by duration, the following calculation will need to be made:

$$\text{convexity effect} = 1/2 \times \text{convexity} \times \Delta y^2$$

The convexity effect is typically quite small. However, remember that convexity is simply correcting for the error embedded in the duration, so you would expect convexity to have a much smaller effect than duration. Also note that for an option-free bond, the convexity effect is always positive, no matter which direction interest rates move. Thus, for option-free bonds, convexity is always added to duration to modify the price volatility errors embedded in duration. This decreases the drop in price (due to an increase in yields) and adds to the rise in price (due to a fall in yields).

Now, by combining duration and convexity, we can obtain a far more accurate estimate of the percentage change in the price of a bond, especially for large swings in yield. That is, you can account for the amount of convexity embedded in a bond by adding the convexity effect to the duration effect as follows:

Example: Estimating price changes with the duration/convexity approach

Estimate the effect of a 100 basis point increase and decrease on a 10-year, 5%, option-free bond currently trading at par, using the duration/convexity approach. The bond has a duration of 7 and a convexity of 90.

Answer:

Using the duration/convexity approach:

$$\text{percentage bond price change} \approx \text{duration effect} + \text{convexity effect}$$

$$\Delta B_{+\Delta y} \approx [-7 \times 0.01] + [(1/2) \times 90 \times (0.01^2)]$$

$$\approx -0.07 + 0.0045 = -0.0655 = -6.55\%$$

$$\Delta B_{-\Delta y} \approx [-7 \times -0.01] + [(1/2) \times 90 \times (-0.01^2)]$$

$$\approx 0.07 + 0.0045 = 0.0745 = 7.45\%$$

Theories of the Term Structure

AIM 26.8: Define and discuss the major theories of the term structure of interest rates.

The **expectations theory** suggests that forward rates correspond to expected future spot rates. That is, forward rates are good predictors of expected future spot rates. In reality, the expectations theory fails to explain all future spot rate expectations. The **market segmentation theory** states that the bond market is segmented into different maturity sectors and that supply and demand for bonds in each maturity range dictate rates in that maturity range. The **liquidity preference theory** suggests that most depositors prefer short-term liquid deposits. In order to coax them to lend longer term, the intermediary will raise longer-term rates by adding a liquidity premium.

Key Concepts

1. Three types of interest rates are particularly relevant in the interest rate derivative markets: Treasury rates, London Interbank Offered Rate (LIBOR), and repo rates.

2. Derivative valuation models use the continuous compounding assumption due to its elegant mathematical properties.

3. Zero or spot rates correspond to the interest earned on a single cash flow at a single point in time. Bond prices are computed using the spot curve by discounting each cash flow at the appropriate spot rate.

4. The yield of a bond is the single discount rate that equates the present value of a bond to its market price.

5. Zero rates are computed using the bootstrapping methodology. Forward rates are computed from spot rates.

6. When the spot curve is upward-sloping, the corresponding forward rate curve is upward-sloping and above the spot curve. When the spot curve is downward-sloping, the corresponding forward rate curve is downward-sloping and below the spot curve.

7. A forward-rate agreement is a contract between two parties that an interest rate will apply to a specific principal during some future time period.

8. Duration and modified duration are the same when continuously compounded yields are used, and they both estimate the percentage price change of a bond from an absolute change in yield.

9. The expectations theory suggests that forward rates correspond to expected future spot rates. The market segmentation theory states that bonds are segmented into different maturity sectors and that supply and demand dictate rates in the segmented maturity sectors. The liquidity preference theory suggests that longer-term rates incorporate a liquidity premium.

Concept Checkers

1. What is the continuously compounded rate of return for an investment that has a value today of $86.50 and will have a future value of $100 in one year?
   A. 13.62%.
   B. 14.50%.
   C. 15.61%.
   D. 16.38%.

2. Assume that the continuously compounded 10-year spot rate is 5% and the 9-year spot rate is 4.9%. Which of the following is closest to the 1-year forward rate nine years from now?
   A. 4.1%.
   B. 5.1%.
   C. 5.9%.
   D. 6.0%.

3. An investor enters into a 1-year forward rate agreement (FRA) where she will receive the contracted rate on a principal of $1 million. The contracted rate is a 1-year rate at 5%. Which of the following is closest to the cash flow if the actual rate is 6% at maturity of the underlying asset (loan)?
   A. –$10,000.
   B. –$1,000.
   C. +$1,000.
   D. +$10,000.

4. What is the bond price of a $100 face value, 2.5-year, 3% semiannual coupon bond using the following annual continuously compounded spot rates: z_1 = 3%, z_2 = 3.1%, z_3 = 3.2%, z_4 = 3.3%, and z_5 = 3.4%?
   A. $97.27.
   B. $97.83.
   C. $98.15.
   D. $98.99.

5. A $100 face value, 1-year, 4% semiannual bond is priced at 99.806128. If the annualized 6-month spot rate (z_1) is 4.1%, what is the 1-year spot rate (z_2)? (Both spots are continuously compounded rates.)
   A. 4.07%.
   B. 4.16%.
   C. 4.20%.
   D. 4.26%.

Concept Checker Answers

1. **B** The formula to solve this problem is:

$$R_c = m \times \ln\left(1+\frac{R}{m}\right)$$

First, we need to compute *R* as the rate earned on the \$86.50 investment:

$$R = \frac{\$100 - \$86.50}{\$86.50} = 0.15607$$

This is essentially the effective rate earned over one year with annual compounding. So, m = 1, and $R_c = 1 \times \ln(1.15607) = 0.1450$. Alternatively, since m = 1,

$$\ln\left(\frac{100}{86.50}\right) = 0.1450 = 14.50\%$$

2. **C** $R_{Forward} = R_2 + (R_2 - R_1) \times [T_1 / (T_2 - T_1)] = 0.05 + (0.05 - 0.049) \times [9 / (10 - 9)] = 5.9\%$

3. **A** \$1,000,000 (0.05 – 0.06)(1) = –\$10,000

4. **D** $B = 1.5 \times e^{[(-0.03/2)\times 1]} + 1.5 \times e^{[(-0.031/2)\times 2]} + 1.5 \times e^{[(-0.032/2)\times 3]} + 1.5 \times e^{[(-0.033/2)\times 4]} + 101.5 \times e^{[(-0.034/2)\times 5]} = 1.48 + 1.45 + 1.43 + 1.40 + 93.23 = \$98.99$

5. **B** $B = 2 \times e^{[(-z_1/2)\times 1]} + 102 \times e^{[(-z_2/2)\times 2]}$; $\$99.806128 = 2 \times e^{[(-0.041/2)\times 1]} + 102 \times e^{[(-z_2/2)\times 2]}$; $\$97.846711 = 102 \times e^{[(-z_2/2)\times 2]}$; $z_2 = 0.0415707 = 4.16\%$

The following is a review of the Financial Markets and Products principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Determination of Forward and Futures Prices

Topic 27

Exam Focus

Both forward and futures contracts are obligations regarding a future transaction. Because the difference in pricing between these contract types is small, forward contract pricing and futures contract pricing are often presented interchangeably. The basic model for forward prices is the cost-of-carry model, which essentially connects the forward price to the cost incurred from purchasing and storing the underlying asset until the contract maturity date. Cash flows over the life of the contract are easily incorporated into the pricing model. Futures contracts contain delivery options that benefit the short seller of the contract. These delivery options must be incorporated into the futures pricing model.

Investment and Consumption Assets

AIM 27.1: **Differentiate between investment and consumption assets.**

An **investment asset** is an asset that is held for the purpose of investing. This type of asset is held by many different investors for the sake of investment. Examples of investment assets include stocks and bonds. A **consumption asset** is an asset that is held for the purpose of consumption. Examples of consumption assets include commodities such as oil and natural gas.

Short-Selling and Short Squeeze

AIM 27.2: **Define short-selling and short squeeze.**

Short sales are orders to sell securities that the seller does not own. Short selling is also known as "shorting" and is possible with some investment assets. For a short sale, the short seller (1) simultaneously borrows and sells securities through a broker, (2) must return the securities at the request of the lender or when the short sale is closed out, and (3) must keep a portion of the proceeds of the short sale on deposit with the broker.

The short seller may be forced to close his position if the broker runs out of securities to borrow. This is known as a **short squeeze**, and the seller will need to close his short position immediately.

Why would anyone ever want to sell securities short? The seller thinks the current price is too high and that it will fall in the future, so the short seller hopes to sell high and then buy low. If a short sale is made at \$30 per share and the price falls to \$20 per share, the short seller can buy shares at \$20 to replace the shares borrowed and keep \$10 per share as profit.

Two rules currently apply to short selling:

1. The short seller must pay all dividends due to the lender of the security.
2. The short seller must deposit collateral to guarantee the eventual repurchase of the security.

Forward and Futures Contracts

AIM 27.3: Discuss the differences between forward and futures contracts and explain the relationship between forward and spot prices.

AIM 27.4: Calculate the forward price, given the underlying asset's price, with or without short sales and/or consideration to the income or yield of the underlying asset. Describe an arbitrage argument in support of these prices.

AIM 27.9: Define and calculate, using the cost-of-carry model, forward prices where the underlying asset either does or does not have interim cash flows.

Futures contracts and **forward contracts** are *similar* in that both:

- Can be either deliverable or cash settlement contracts.
- Are priced to have zero value at the time an investor enters into the contract.

Futures contracts *differ* from forward contracts in the following ways:

- Futures contracts trade on organized exchanges. Forwards are private contracts and do not trade on an exchange.
- Futures contracts are highly standardized. Forwards are customized contracts satisfying the needs of the parties involved.
- A single clearinghouse is the counterparty to all futures contracts. Forwards are contracts with the originating counterparty.
- The government regulates futures markets. Forward contracts are usually not regulated.

Forward Prices

The pricing model used to compute forward prices makes the following assumptions:

- No transaction costs or short-sale restrictions.
- Same tax rates on all net profits.
- Borrowing and lending at the risk-free rate.
- Arbitrage opportunities are exploited as they arise.

For the development of a forward pricing model, we will use the following notation:

- T = time to maturity (in years) of the forward contract.
- S_0 = underlying asset price today ($t = 0$).
- F_0 = forward price today.
- r = continuously compounded risk-free annual rate.

The forward price may be written as:

Equation 1

$$F_0 = S_0e^{rT}$$

The right-hand side of Equation 1 is the cost of borrowing funds to buy the underlying asset and carrying it forward to time *T*. Equation 1 states that this cost must equal the forward price. If $F_0 > S_0e^{rT}$, then arbitrageurs will profit by selling the forward and buying the asset with borrowed funds. If $F_0 < S_0e^{rT}$, arbitrageurs will profit by selling the asset, lending out the proceeds, and buying the forward. Hence, the equality in Equation 1 must hold. Note that this model assumes perfect markets.

As it turns out, actual short sales are not necessary for Equation 1 to hold. All that is necessary is a sufficient number of investors who are not only holding the investment asset but also are willing to sell the asset if the forward price becomes too low. In the event that the forward price is too low, the investor will sell the asset and take a long position in the forward contract. This is important since the arbitrage relationship in Equation 1 must hold for all investment assets even though short selling is not available for every asset.

Example: Computing a forward price with no interim cash flows

Suppose we have an asset currently worth $1,000. The current continuously compounded rate is 4% for all maturities. Compute the price of a 6-month forward contract on this asset.

Answer:

$$F_0 = \$1{,}000e^{0.04(0.5)} = \$1{,}020.20$$

Forward Price With Carrying Costs

If the underlying pays a known amount of cash over the life of the forward contract, a simple adjustment is made to Equation 1. Since the owner of the forward contract does not receive any of the cash flows from the underlying asset between contract origination and delivery, the present value of these cash flows must be deducted from the spot price when calculating the forward price. This is most easily seen when the underlying asset makes a periodic payment. With this in mind, we let *I* represent the *present value* of the cash flows over *T* years. Equation 1 then becomes:

Equation 2

$$F_0 = (S_0 - I)\, e^{rT}$$

The same arbitrage arguments used for Equation 1 are used here. The only modification is that the arbitrageur must account for the known cash flows.

Example: Forward price when underlying asset has a cash flow

Compute the price of a 6-month forward on a coupon bond that pays a 5% coupon semiannually. A coupon is to be paid in three months. Assume the risk-free rate is 4%.

Answer:

The cost of carry (income) in this case is computed as:

$$I = 25e^{-0.04(0.25)} = \$24.75125$$

Using Equation 2:

$$F_0 = (\$1{,}000 - \$24.75125)e^{0.04(0.5)} = \$994.95$$

The Effect of a Known Dividend

When the underlying asset for a forward contract pays a dividend, we assume that the dividend is paid continuously. Letting q represent the continuously compounded dividend yield paid by the underlying asset expressed on a per annum basis, Equation 1 becomes:

Equation 3

$$F_0 = S_0e^{(r-q)T}$$

Once again, the same arbitrage arguments are used to prove that Equation 3 must be true.

Example: Forward price when the underlying asset pays a dividend

Compute the price of a 6-month forward contract for which the underlying asset is a stock index with a value of \$1,000 and a continuous dividend yield of 1%. Assume the risk-free rate is 4%.

Answer:

Using Equation 3:

$$F_0 = \$1{,}000e^{(0.04-0.01)0.5} = \$1{,}015.11$$

Value of a Forward Contract

The initial value of a forward contract is zero. After its inception, the contract can have a positive value to one counterparty (and a negative value to the other). Since the forward price at every moment in time is computed to prevent arbitrage, the value at inception of the contract must be zero. The forward contract can take on a non-zero value only after the contract is entered into and the obligation to buy or sell has been made. If we denote the obligated delivery price after inception as *K*, then the value of the long contract on an asset with no cash flows is computed as $S_0 - Ke^{-rT}$; with cash flows (with present value *I*) it is $S_0 - I - Ke^{-rT}$; and with a continuous dividend yield of *q*, it is $S_0e^{-qT} - Ke^{-rT}$.

Example: Value of a stock index forward contract

Using the stock index forward in the previous example, compute the value of a long position if the index increases to $1,050 immediately after the contract is purchased.

Answer:

In this case, $K = \$1{,}015.11$ and $S_0 = \$1{,}050$, so the value is:

$$\$1{,}050e^{-0.01(0.5)} - \$1{,}015.11e^{-0.04(0.5)} = \$49.75$$

Currency Futures

AIM 27.6: Use the interest rate parity relationship to calculate a forward foreign exchange rate.

Interest rate parity (IRP) states that the forward exchange rate, *F* (measured in domestic per unit of foreign currency), must be related to the spot exchange rate, *S*, and to the interest rate differential between the domestic and the foreign country, $r - r_f$.

The general form of the interest rate parity condition is expressed as:

$$F = Se^{(r-r_f)T}$$

This equation is a no-arbitrage relationship. Using our notation from earlier, we can state the interest rate parity relationship as:

Equation 4

$$F_0 = S_0e^{(r-r_f)T}$$

Note that this is equivalent to Equation 3 with r_f replacing *q*. Just as the continuous dividend yield *q* was used to adjust the cost of carry, we use the continuous yield on a foreign currency deposit here.

Example: Currency futures pricing

Suppose we wish to compute the futures price of a 10-month futures contract on the Mexican peso. Each contract controls 500,000 pesos and is quoted in terms of dollar/peso. Assume that the continuously compounded risk-free rate in Mexico (r_f) is 14%, the continuously compounded risk-free rate in the United States is 2%, and the current exchange rate is 0.12.

Answer:

Applying Equation 4:

$$F_0 = \$0.12e^{(0.02-0.14)\frac{10}{12}} = \$0.10858/\text{peso}$$

Professor's Note: The concept of interest rate parity will show up again in the foreign exchange risk topic.

Forward Prices vs. Futures Prices

AIM 27.5: Explain the relationship between forward and futures prices.

The most significant difference between forward contracts and futures contracts is the daily marking to market requirement on futures contracts. When interest rates are known over the life of a contract, *T*, forward and futures prices can be shown to be the same. Various relationships can be derived, depending on the assumptions made between the value of the underlying and the level of change in interest rates. In general, when *T* is small, the price differences are usually very small and can be ignored. Empirical research comparisons of forwards and futures prices are mixed. Some studies conclude a significant difference and others do not. The important concept to understand here is that assuming the two are the same is an approximation, and under certain circumstances the approximation can be inaccurate.

Commodity Futures

AIM 27.7: Define income, storage costs, and convenience yield.

AIM 27.8: Calculate the futures price on commodities incorporating income/ storage costs and/or convenience yields.

Professor's Note: There are two topics in the FRM curriculum related to commodity forwards and futures (Topics 32 and 33). In those topics, you will learn more about storage costs and the convenience yield as well as the arbitrage relationships that must hold with commodity futures.

Income and Storage Costs

When the underlying is considered a consumption asset, the pricing relationships developed above do not adequately capture all the necessary characteristics of the asset. *Consumption assets have actual storage costs associated with them.* These costs increase the carrying costs. The costs can be expressed either as a known cash flow or as a yield. Let *U* denote the present value of known storage cost over the life of the forward contract. Equation 1 then becomes:

Equation 5

$$F_0 = (S_0 + U)e^{rT}$$

If we express the storage costs in terms of a continuous yield, *u*:

Equation 6

$$F_0 = S_0 e^{(r+u)T}$$

The arbitrage relationships are the same except we need to account for the additional carrying costs over *T* years. However, when the owner of these assets is reluctant to sell the asset, Equations 5 and 6 are replaced by:

Equation 7

$$F_0 \leq (S_0 + U)e^{rT}$$

And:

Equation 8

$$F_0 \leq S_0 e^{(r+u)T}$$

Convenience Yield

Equations 7 and 8 suggest there is a *benefit to owning the underlying consumable asset compared to owning the future.* If we introduce a **convenience yield**, *y*, to balance Equations 7 and 8, we have:

$$F_0 e^{yT} = (S_0 + U)e^{rT} = S_0 e^{(r+u)T}$$

This formula can be reduced to:

Equation 9

$$F_0 = S_0 e^{(r+u-y)T}$$

In other words, the convenience yield is simply the yield required to produce an equality and is thus a measure of the benefit of owning spot, or physical, consumption commodities.

Delivery Options in the Futures Market

AIM 27.10: Discuss the various delivery options available in the futures markets and how they can influence futures prices.

Some futures contracts grant **delivery options** to the short—options on what, where, and when to deliver. Some Treasury bond contracts give the short a choice of several bonds that are acceptable to deliver and options as to when to deliver during the expiration month. Physical assets, such as gold or corn, may offer a choice of delivery locations to the short. These options can be of significant value to the holder of the short position in a futures contract.

As shown in the previous discussion on commodity futures, if the cost of carrying the asset is greater than the convenience yield (benefit from holding the physical asset), it is ideal for the short position to deliver the contract early. This scenario suggests that the futures price will increase over time; hence, the short has an incentive to deliver early. The opposite relationship holds true when the cost of carry is less than the convenience yield. In this case, the short position will delay delivery since the futures price is expected to fall over time.

Futures and Expected Future Spot Prices

AIM 27.11: Analyze the relationship between current futures prices and expected future spot prices, including the impact of systematic and nonsystematic risk.

The cost of carry model is a widely used method for estimating the appropriate price of a futures contract, but other theories exist for explaining the futures price. One intuitively appealing model expresses the futures price as a function of the expected spot price (S_T).

$$F_0 = E(S_T)$$

For obvious reasons, this is called the **expectations model** and states that the current futures price for delivery at time T is equal to the expected spot price at time T. Similar to the no-arbitrage rule, this model acts to keep the current futures price in line with the expected spot rate at that time. If the futures price is less than the expected price, aggressive buying of the futures would push up the futures price. If the futures price is greater than the expected spot rate, aggressive selling of the futures would lead to lower the futures price. Although intuitively appealing, other factors probably play a role in the pricing mechanism. Indeed,

if the expectations model limited traders to a risk-free rate of return, there would be no incentive to buy or sell contracts.

Cost of Carry vs. Expectations

Economist John Maynard Keynes found the expectations model to be flawed precisely because it provided no justification for speculators to enter the market. Futures contracts provide a mechanism to transfer risk from those who need to hedge their positions (e.g., farmers who are long the commodities) to speculators. In order to entice speculators to bear the risk of these contracts, there has to exist an expectation of profit greater than the risk-free rate. For this to occur, the futures contract price must be less than the expected spot rate at maturity [$F_0 < E(S_T)$] and must continually increase during the term of the contract. Keynes referred to this as **normal backwardation.** This relationship suggests that the asset underlying the futures contract exhibits positive systematic risk, since this is the risk that remains after diversifying away all nonsystematic risk.

On the other side of the contracts are those who are users of the commodity who want to shift some of the risk of rising market prices to speculators. They wish to purchase futures contracts from speculators. The speculators have to be enticed into assuming this risk by the expectation of profits that would exceed the risk-free rate. From this perspective, the futures price must be higher than the expected spot price at maturity [$F_0 > E(S_T)$] and must continually decrease during the term of the contract. Keynes referred to this expectation as **contango** (a.k.a. normal contango). This relationship suggests that the asset underlying the futures contract exhibits negative systematic risk.

Contango and Backwardation

AIM 27.12: Define contango and backwardation, interpret the effect contango or backwardation may have on the relationship between commodity futures and spot prices, and relate the cost-of-carry model to contango and backwardation.

Backwardation refers to a situation where the futures price is below the spot price. For this to occur, there must be a significant benefit to holding the asset. Backwardation might occur if there are benefits to holding the asset that offset the opportunity cost of holding the asset (the risk-free rate) and additional net holding costs.

Contango refers to a situation where the futures price is above the spot price. If there are no benefits to holding the asset (e.g., dividends, coupons, or convenience yield), contango will occur because the futures price will be greater than the spot price.

Professor's Note: In this case, the reference to backwardation and contango refers to the relationship between the futures price and the current spot price, not the expected spot price.

Key Concepts

1. Short sales are orders to sell securities that the seller does not own. A short squeeze results if the broker runs out of securities to borrow.

2. Forward and futures contracts are similar because they are both future obligations to transact an asset on some future date.

3. Forward contracts do not trade on an exchange, are not standardized, and do not normally close out prior to expiration.

4. The cost-of-carry model is used to price forward and futures contracts. It states that the total cost of carrying the underlying asset to expiration must be the futures price. Any other price results in arbitrage.

5. The futures price or cost-of-carry model is easily accommodated for interim cash flows from the underlying asset.

6. Nonfinancial futures contracts are more difficult to evaluate due to consumption value and convenience yields.

7. Contango is the situation in which the futures price is above the current spot price. Backwardation is the opposite relationship.

Concept Checkers

Use the following data to answer Questions 1 and 2.

An investor has an asset that is currently worth $500, and the continuously compounded risk-free rate at all maturities is 3%.

1. Which of the following is the closest to the no-arbitrage price of a 3-month forward contract?
   A. $496.26.
   B. $500.00.
   C. $502.00.
   D. $503.76.

2. If the asset pays a continuous dividend of 2%, which of the following is the closest to the no-arbitrage price of a 3-month forward contract?
   A. $494.24.
   B. $498.75.
   C. $501.25.
   D. $506.29.

3. A bond pays a semiannual coupon of $40 and has a current value of $1,109. The next payment on the bond is in four months and the interest rate is 6.50%. Using the continuous time model, the price of a 6-month forward contract on this bond is closest to:
   A. $995.62.
   B. $1,011.14.
   C. $1,035.65.
   D. $1,105.20.

4. The owner of 300,000 bushels of corn wishes to hedge his position for a sale in 150 days. The current price of corn is $1.50/bushel and the contract size is 5,000 bushels. The interest rate is 7%, compounded daily. The storage cost for the corn is $18/day. Assume the cost of storage as a percentage of the contract per year is 1.46%. The price for the appropriate futures contract used to hedge the position is closest to:
   A. $6,635.
   B. $7,248.
   C. $7,656.
   D. $7,765.

5. Backwardation refers to a situation where:
   A. spot prices are above futures prices.
   B. spot prices are below futures prices.
   C. expected future spot prices are above futures prices.
   D. expected future spot prices are below futures prices.

Concept Checker Answers

1. D Using Equation 1:

   $500e^{(0.03)\,(0.25)} = \$503.76$

   where S = 500, T = 0.25, and r = 0.03

2. C Using Equation 3:

   $500e^{(0.03-0.02)0.25} = \$501.25$

3. D Use the formula $F_0 = (S_0 - I)e^{rT}$, where *I* is the present value of \$40 to be received in 4 months, or 0.333 years. At a discount rate of 6.50%:

   $I = \$40 \times e^{-0.065 \times 0.333} = \$39.14$

   $F_0 = (\$1{,}109 - 39.14) \times e^{(0.065 \times 0.5)} = \$1{,}105.20$

4. D Since both the interest and the storage costs compound on a daily basis, a continuous time model is appropriate to approximate the price of the contract.

   The cost of storage as a percentage of the contract per year is:

   $$u = 365 \times \frac{18}{1.50 \times 300{,}000} = 0.0146$$

   Using Equation 6, the futures price per bushel is:

   $F = \$1.50 \times e^{(0.07 + 0.0146)(150/365)} = \$1.553 \times 5{,}000$ bushels per contract = \$7,765.34

5. A Backwardation refers to a situation where spot prices are higher than futures prices. Significant monetary benefits of the asset or a relatively high convenience yield can lead to this result.

The following is a review of the Financial Markets and Products principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Interest Rate Futures

Topic 28

Exam Focus

In this topic, we examine Treasury bonds (T-bonds) and Eurodollar futures contracts. These instruments are two of the most popular interest rate futures contracts that trade in the United States. Be able to define the cheapest-to-deliver bond for T-bonds and know how to use the convexity adjustment for Eurodollar futures. Duration-based hedging using interest rate futures is also discussed. Be familiar with the equation to calculate the number of contracts needed to conduct a duration-based hedge.

Day Count Conventions

AIM 28.1: List the most commonly used day count conventions, identify the markets that each one is typically used in, and apply each to an interest calculation.

Day count conventions play a role when computing the interest that accrues on a fixed income security. When a bond is purchased, the buyer must pay any **accrued interest** earned through the settlement date.

$$\text{accured interest} = \text{coupon} \times \frac{\#\text{ of days from last coupon to the settlement date}}{\#\text{ of days in coupon period}}$$

In the United States, there are three commonly used day count conventions.

1. U.S. Treasury bonds use **actual/actual**.
2. U.S. corporate and municipal bonds use **30/360**.
3. U.S. money market instruments (Treasury bills) use **actual/360**.

The following examples demonstrate the use of day count conventions when computing accrued interest.

Example: Day count conventions

Suppose there is a semiannual-pay bond with a $100 par value. Further assume that coupons are paid on March 1 and September 1 of each year. The annual coupon is 6%, and it is currently July 13. **Compute** the accrued interest of this bond as a T-bond and a U.S. corporate bond.

Answer:

The T-bond uses actual/actual (in period), and the reference (March 1 to September 1) period has 184 days. There are 134 actual days from March 1 to July 13, so the accrued interest is:

$$\frac{134}{184} \times \$3 = \$2.1848$$

The corporate bond uses 30/360, so the reference period now has 180 days. Using this convention, there are 132 (= 30 × 4 + 12) days from March 1 to July 13, so the accrued interest is:

$$\frac{132}{180} \times \$3 = \$2.20$$

Quotations for T-Bonds

AIM 28.3: Differentiate between the clean and dirty price for a U.S. Treasury bond; calculate the accrued interest and dirty price on a U.S. Treasury bond.

T-bond prices are quoted relative to a $100 par amount in dollars and 32nds. So a 95–05 is 95 5/32, or 95.15625. The quoted price of a T-bond is not the same as the cash price that is actually paid to the owner of the bond. In general:

cash price = quoted price + accrued interest

Clean and Dirty Prices

The cash price (a.k.a. **dirty price**) is the price that the seller of the bond must be paid to give up ownership. It includes the present value of the bond (a.k.a. quoted price and clean price) plus the accrued interest. This relationship is shown in the equation above. Conversely, the **clean price** is the cash price less accrued interest:

quoted price = cash price – accrued interest

This relationship can also be expressed as:

clean price = dirty price – accrued interest

Example: Calculate the cash price of a bond

Assume the bond in the previous example is a T-bond currently quoted at 102–11. **Compute** the cash price.

Answer:

cash price = \$102.34375 + \$2.1848 = \$104.52855

For a \$100,000 par amount, this is \$104,528.55.

Quotations for T-Bills

AIM 28.2: Convert from a discount rate to a price for a U.S. Treasury bill.

T-bills and other money-market instruments use a discount rate basis and an actual/360 day count. A T-bill with a \$100 face value with n days to maturity and a cash price of Y is quoted as:

$$\text{T-bill discount rate} = \frac{360}{n}(100 - Y)$$

This is referred to as the discount rate in annual terms. However, this discount rate is not the actual rate earned on the T-bill. The following example shows the calculation of the annualized yield on a T-bill, given its price.

Example: Calculating the cash price on a T-Bill

Suppose you have a 180-day T-bill with a discount rate, or quoted price, of five (i.e., the annualized rate of interest earned is 5% of face value). If face value is \$100, what is the true rate of interest and the cash price?

Answer:

Interest is equal to \$2.5 (= \$100 × 0.05 × 180 / 360) for a 180-day period. The true rate of interest for the period is therefore 2.564% [= 2.5 / (100 – 2.5)].

Cash price: 5 = (360 / 180) × (100 – Y); Y = \$97.5.

Treasury Bond Futures

AIM 28.4: Explain and calculate a U.S. Treasury bond futures contract conversion factor.

AIM 28.5: Calculate the cost of delivering a bond into a Treasury bond futures contract.

AIM 28.6: Describe the impact of the level and shape of the yield curve on the cheapest-to-deliver bond decision.

In a T-bond futures contract, any government bond with more than 15 years to maturity on the first of the delivery month (and not callable within 15 years) is deliverable on the contract. This produces a large supply of potential bonds that are deliverable on the contract and reduces the likelihood of market manipulation. Since the deliverable bonds have very different market values, the Chicago Board of Trade (CBOT) has created **conversion factors**. The conversion factor defines the price received by the short position of the contract (i.e., the short position is delivering the contract to the long). Specifically, the cash received by the short position is computed as follows:

cash received = (QFP × CF) + AI

where:
QFP = quoted futures price
CF = conversion factor for the bond delivered
AI = accrued interest since the last coupon date on the bond delivered

Conversion factors are supplied by the CBOT on a daily basis. Conversion factors are calculated as the discounted price of a bond minus any accrued interest all divided by face value. For example, if the present value of a bond is \$142, accrued interest is \$2, and face value is \$100, the conversation factor would be: (142 – 2) / 100 = 1.4.

Cheapest-to-Deliver Bond

The conversion factor system is not perfect and often results in one bond that is the cheapest (or most profitable) to deliver. The procedure to determine which bond is the cheapest-to-deliver (CTD) is as follows:

cash received by the short = (QFP × CF) + AI

cost to purchase bond = (quoted bond price + AI)

The CTD is the bond that minimizes the following: quoted bond price – (QFP × CF). This formula calculates the cost of delivering the bond.

Example: The cheapest-to-deliver bond

Assume an investor with a short position is about to deliver a bond and has four bonds to choose from which are listed in the following table. The last settlement price is \$95.75 (this is the quoted futures price). Determine which bond is the cheapest-to-deliver.

Bond	*Quoted Bond Price*	*Conversion Factor*
1	99	1.01
2	125	1.24
3	103	1.06
4	115	1.14

Answer:

Cost of delivery:

Bond 1: 99 – (95.75 × 1.01) = \$2.29

Bond 2: 125 – (95.75 × 1.24) = \$6.27

Bond 3: 103 – (95.75 × 1.06) = \$1.51

Bond 4: 115 – (95.75 × 1.14) = \$5.85

Bond 3 is the cheapest-to-deliver with a cost of delivery of \$1.51.

Finding the cheapest-to-deliver bond does not require any arcane procedures but could involve searching among a large number of bonds. The following guidelines give an indication of what type of bonds tend to be the cheapest-to-deliver under different circumstances:

- When yields > 6%, CTD bonds tend to be low-coupon, long-maturity bonds.
- When yields < 6%, CTD bonds tend to be high-coupon, short-maturity bonds.
- When the yield curve is upward sloping, CTD bonds tend to have longer maturities.
- When the yield curve is downward sloping, CTD bonds tend to have shorter maturities.

Treasury Bond Futures Price

AIM 28.7: Calculate the theoretical futures price for a Treasury bond futures contract.

Recall the cost-of-carry relationship, where the underlying asset pays a known cash flow, as was presented in the previous topic. The futures price is calculated in the following fashion:

$$F_0 = (S_0 - I)e^{rT}$$

where:
I = present value of cash flow

We can use this equation to calculate the theoretical futures price when accounting for the CTD bond's accrued interest and its conversion factor.

Example: Theoretical futures price

Suppose that the CTD bond for a Treasury bond futures contract pays 10% semiannual coupons. This CTD bond has a conversion factor of 1.1 and a quoted bond price of 100. Assume that there are 180 days between coupons and the last coupon was paid 90 days ago. Also assume that Treasury bond futures contract is to be delivered 180 days from today, and the risk-free rate of interest is 3%. **Calculate** the theoretical price for this T-bond futures contract.

Answer:

The cash price of the CTD bond is equal to the quoted bond price plus accrued interest. Accrued interest is computed as follows:

$$\text{AI} = \text{coupon} \times \left(\frac{\text{number of days from last coupon to settlement date}}{\text{number of days in coupon period}} \right)$$

$$\text{AI} = 5 \times \frac{90}{180} = 2.5$$

$$\text{cash price} = 100 + 2.5 = 102.5$$

Since the next coupon will be received 90 days from today, that cash flow should be discounted back to the present using the familiar present value equation which discounts the cash flow using the risk-free rate:

$$5e^{-0.03 \times (90/365)} = \$4.96$$

Using the cost-of-carry model, the cash futures price (which expires 180 days from today) is then calculated as follows:

$$F_0 = (102.5 - 4.96)e^{(0.03)(180/365)} = 98.99$$

We are not done, however, since the futures contract expires 90 days after the last coupon payment. The quoted futures price at delivery is calculated after subtracting the amount of accrued interest (recall: QFP = cash futures price – AI).

$$98.99 - \left(5 \times \frac{90}{180}\right) = \$96.49$$

Finally, the conversion factor is utilized, producing a theoretical price for this T-bond futures contract of:

$$\text{QFP} = \frac{96.49}{1.1} = \$87.72$$

Eurodollar Futures

AIM 28.8: Calculate the final contract price on a Eurodollar futures contract.

AIM 28.9: Describe and compute the Eurodollar futures contract convexity adjustment.

The 3-month **eurodollar futures** contract trades on the Chicago Mercantile Exchange (CME) and is the most popular interest rate futures in the United States. This contract settles in cash and the minimum price change is one "tick," which is a price change of one basis point, or $25 per $1 million contract. Eurodollar futures are based on a eurodollar deposit (a eurodollar is a U.S. dollar deposited outside the United States) with a face amount of $1 million. The interest rate underlying this contract is essentially the 3-month (90-day) forward LIBOR. If *Z* is the quoted price for a eurodollar futures contract, the contract price is:

eurodollar futures price = $10,000[100 – (0.25)(100 – Z)]

For example, if the quoted price, *Z*, is 97.8:

contract price = $10,000[100 – (0.25)(100.0 – 97.8)] = $994,500

Convexity Adjustment

The corresponding 90-day forward LIBOR (on an annual basis) for each contract is 100 – Z. For example, assume that the previous eurodollar contract was for a futures contract that matured in six months. Then the 90-day forward LIBOR six months from now is approximately 2.2% (100 – 97.8). However, the daily marking to market aspect of the futures contract can result in differences between actual forward rates and those implied by futures contracts. This difference is reduced by using the convexity adjustment. In general, long-dated eurodollar futures contracts result in implied forward rates larger than actual forward rates. The two are related as follows:

actual forward rate = forward rate implied by futures – ($\frac{1}{2} \times \sigma^2 \times T_1 \times T_2$)

where:
T_1 = the maturity on the futures contract
T_2 = the time to the maturity of the rate underlying the contract (90 days)
σ = the annual standard deviation of the change in the rate underlying the futures contract, or 90-day LIBOR

Notice that as T_1 increases, the convexity adjustment will need to increase. So as the maturity of the futures contract increases, the necessary convexity adjustment increases. Also, note that the σ and the T_2 are largely dictated by the specifications of the futures contract.

AIM 28.10: Demonstrate how Eurodollar futures can be used to extend the LIBOR zero curve.

Forward rates implied by convexity-adjusted eurodollar futures can be used to produce a LIBOR spot curve (also called a LIBOR zero curve since spot rates are sometimes referred to as zero rates). Recall the equation presented previously in Topic 26, which was used to generate the shape of the *futures* rate curve:

$$R_{Forward} = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

where:
R_i = spot rate corresponding with T_i periods
$R_{Forward}$ = the forward rate between T_1 and T_2

This forward rate equation can be rearranged to solve for the *spot* rate for the next time period (T_2):

$$R_2 = \frac{R_{Forward}(T_2 - T_1) + R_1 T_1}{T_2}$$

Given the first LIBOR spot rate (R_1) and the length of each forward contract period, we can calculate the next spot rate (R_2). The rate at T_2 can then be used to find the rate at T_3 and so on. The end result is a generated LIBOR spot (zero) curve.

Duration-Based Hedging

AIM 28.11: Calculate the duration-based hedge ratio and describe a duration-based hedging strategy using interest rate futures.

The objective of a **duration-based hedge** is to create a combined position that does not change in value when yields change by a small amount. In other words, a position that has a duration of zero needs to be produced. The combined position consists of our portfolio with a hedge horizon value of P and a futures position with a contract value of F. Denote the duration of the portfolio at the hedging horizon as D_P and the corresponding duration of the futures contract as D_F. Using this notation, the duration-based hedge ratio can be expressed as follows:

$$N = -\frac{P \times D_P}{F \times D_F}$$

where:
N = number of contracts to hedge

The minus sign suggests that the futures position is the opposite of the original position. In other words, if the investor is long the portfolio, he must short N contracts to produce a position with a zero duration.

Example: Duration-based hedge

Assume there is a 6-month hedging horizon and a portfolio value of $100 million. Further assume that the 6-month T-bond contract is quoted at 105–09, with a contract size of $100,000. The duration of the portfolio is 10, and the duration of the futures contract is 12. Outline the appropriate hedge for small changes in yield.

Answer:

$$N = -\frac{100{,}000{,}000 \times 10}{105{,}281.25 \times 12} = -791.53$$

Rounding up to the nearest whole number means the manager should short 792 contracts.

Limitations of Duration

AIM 28.12: Explain the limitations of using a duration-based hedging strategy.

The price/yield relationship of a bond is convex, meaning it is nonlinear in shape. Duration measures are linear approximations of this relationship. Therefore, as the change in yield increases, the duration measures become progressively less accurate. Moreover, duration implies that all yields are perfectly correlated. Both of these assumptions place limitations on the use of duration as a single risk measurement tool. When changes in interest rates are both large and nonparallel (i.e., not perfectly correlated), duration-based hedge strategies will perform poorly.

Key Concepts

1. The most common day count conventions are Actual/Actual, 30/360, and Actual/360.
2. Bonds are quoted on a bond equivalent basis and T-bills on a discount rate basis.
3. T-bond futures use a conversion factor to homogenize the pool of deliverable bonds. However, when the yield curve is not flat, there is a single bond that is the cheapest to deliver.
4. Eurodollar contracts are based on LIBOR and are quoted on a discount rate basis. Long-dated eurodollar contracts must be adjusted for convexity before being used to estimate the corresponding forward rates.
5. Duration can be used to compute the number of futures contracts needed to implement a duration-based hedging strategy. The effectiveness of duration-based hedging strategies is limited when there are large changes in yield or nonparallel shifts in the yield curve.

Concept Checkers

1. Assume a 6-month hedging horizon and a portfolio value of $30 million. Further assume that the 6-month Treasury bond (T-bond) contract is quoted at 100–13, with a contract size of $100,000. The duration of the portfolio is 8, and the duration of the futures contract is 12. Which of the following is closest to the appropriate hedge for small changes in yield?
   A. Long 298 contracts.
   B. Short 298 contracts.
   C. Long 199 contracts.
   D. Short 199 contracts.

2. Which of the following items limits the use of duration as a risk metric?
   I. It assumes the price/yield relationship is linear.
   II. It assumes interest rate volatility is constant.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

3. Consider day count convention and, specifically, the following example: A semiannual bond with $100 face value has a 4% coupon. Today is August 3. Assume coupon dates of March 1 and September 1. Which of the following statements is true?
   A. Corporate bonds accrue more interest in July than T-bonds.
   B. Corporate bonds accrue more interest from March 1 to September 1 than September 1 to March 1.
   C. Corporate bonds accrue more interest than T-bonds for this period (March 1 to August 3).
   D. The T-bond accrued interest is $1.67 for this period (March 1 to August 3).

4. Assume an investor is about to deliver a short bond position and has four options to choose from which are listed in the following table. The settlement price is $92.50 (i.e., the quoted futures price). Determine which bond is the cheapest to deliver.

Bond	*Quoted Bond Price*	*Conversion Factor*
1	98	1.02
2	122	1.27
3	105	1.08
4	112	1.15

   A. Bond 1.
   B. Bond 2.
   C. Bond 3.
   D. Bond 4.

5. Assume the cash price on a 90-T-bill is quoted as 98.75. The discount rate is closest to:
   A. 4%.
   B. 7%.
   C. 6%.
   D. 5%.

Concept Checker Answers

1. D $N = -\dfrac{(\$30{,}000{,}000 \times 8)}{(\$100{,}406.25 \times 12)} = -199$

   The appropriate hedge is to short 199 contracts.

2. A The limitations of duration include: (1) that it is valid for only *small changes in yield*, (2) that it assumes the price/yield relationship is linear, and (3) it assumes that changes in yield are the same across all maturities and risk levels (i.e., they're perfectly correlated).

3. C July accrued T-bond interest is 31/184 = 0.1685; July accrued corporate bond interest is 30/180 = 0.1667. T-bonds accrue 155/184 = 0.8424 × \$2 = \$1.685; C-bonds accrue 153/180 = 0.85 × \$2 = \$1.70.

4. A Cost of delivery:

   Bond 1: 98 – (92.50 × 1.02) = \$3.65

   Bond 2: 122 – (92.50 × 1.27) = \$4.53

   Bond 3: 105 – (92.50 × 1.08) = \$5.10

   Bond 4: 112 – (92.50 × 1.15) = \$5.63

   Bond 1 is the cheapest to deliver with a cost of delivery of \$3.65.

5. D The discount rate on a U.S. T-bill is calculated using the following equation:

$$\text{discount rate} = \frac{360}{n} \times (100 - \text{cash price})$$

$$\text{discount rate} = \frac{360}{90} \times (100 - 98.75) = 5\%$$

The following is a review of the Financial Markets and Products principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

SWAPS

Topic 29

EXAM FOCUS

An interest rate swap is an agreement between two parties to exchange interest payments based on a specified principal over a period of time. In a plain vanilla interest rate swap, one of the interest rates is floating, and the other is fixed. Swaps can be used to efficiently alter the interest rate risk of existing assets and liabilities. A currency swap exchanges interest rate payments in two different currencies. For valuation purposes, swaps can be thought of as a long and short position in two different bonds or as a package of forward rate agreements. Credit risk in swaps cannot be ignored.

MECHANICS OF INTEREST RATE SWAPS

AIM 29.1: Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows.

The most common interest rate swap is the **plain vanilla interest rate swap.** In this swap arrangement, Company X agrees to pay Company Y a periodic fixed rate on a notional principal over the tenor of the swap. In return, Company Y agrees to pay Company X a periodic floating rate on the same notional principal. Both payments are in the same currency. Therefore, only the net payment is exchanged. Most interest rate swaps use the London Interbank Offered Rate (LIBOR) as the reference rate for the floating leg of the swap. Finally, since the payments are based in the same currency, there is no need for the exchange of principal at the inception of the swap. This is why it is called notional principal.

For example, companies X and Y enter into a 2-year plain vanilla interest rate swap. The swap cash flows are exchanged semiannually, and the reference rate is 6-month LIBOR. The LIBOR rates are shown in Figure 1. The fixed rate of the swap is 3.784%, and the notional principal is $100 million. We will compute the cash flows for Company X, the fixed payer of this swap.

Figure 1: 6-Month LIBOR

Beginning of Period	*LIBOR*
1	3.00%
2	3.50%
3	4.00%
4	4.50%
5	5.00%

The first cash flow takes place at the end of period one and uses the LIBOR at the beginning of that same period. In other words, at the beginning of each period, both payments for the end of the period are known. The gross cash flows for the end of the first period for both parties are calculated in the following manner:

floating = \$100 million × 0.03 × 0.5 = \$1.5 million

fixed = \$100 million × 0.03784 × 0.5 = \$1.892 million

Note that 0.5 is the semiannual day count. The net payment for Company X is an outflow of \$0.392 million. Note that we are ignoring the many day-count and business-day conventions associated with swaps. Figure 2 shows the other cash flows.

Figure 2: Swap Cash Flows

End of Period	*LIBOR at Beginning of Period*	*Floating Cash Flow*	*Fixed Cash Flow*	*Net X Cash Flow*
1	3.00%	\$1,500,000	\$1,892,000	–\$392,000
2	3.50%	\$1,750,000	\$1,892,000	–\$142,000
3	4.00%	\$2,000,000	\$1,892,000	\$108,000
4	4.50%	\$2,250,000	\$1,892,000	\$358,000

AIM 29.2: Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows.

Let's continue with companies X and Y. Suppose that X has a 2-year floating-rate liability, and Y has a 2-year fixed-rate liability. After they enter into the swap, interest rate risk exposure from their liabilities has completely changed for each party. X has transformed the floating-rate liability into a fixed-rate liability, and Y has transformed the fixed-rate liability to a floating-rate liability. Note that X pays fixed and receives floating, so X's liability becomes fixed.

Similarly, assume that X has a fixed-rate asset and Y has a floating-rate asset tied to LIBOR. After entering into the swap, X has transformed the fixed-rate asset into a floating-rate asset, and Y has transformed the floating-rate asset into a fixed-rate asset.

Financial Intermediaries

AIM 29.3: Explain the role of financial intermediaries in the swaps market.

AIM 29.4: Describe the role of the confirmation in a swap transaction.

In many respects, swaps are similar to forwards:

- Swaps typically require no payment by either party at initiation.
- Swaps are custom instruments.

- Swaps are not traded in any organized secondary market.
- Swaps are largely unregulated.
- Default risk is an important aspect of the contracts.
- Most participants in the swaps market are large institutions.
- Individuals are rarely swap market participants.

There are swap intermediaries who bring together parties with needs for the opposite side of a swap. Dealers, large banks, and brokerage firms, act as principals in trades just as they do in forward contracts. In many cases, a swap party will not be aware of the other party on the offsetting side of the swap since both parties will likely only transact with the intermediary. **Financial intermediaries**, such as banks, will typically earn a spread of about 3 to 4 basis points for bringing two nonfinancial companies together in a swap agreement. This fee is charged to compensate the intermediary for the risk involved. If one of the parties defaults on its swap payments, the intermediary is responsible for making the other party whole.

Confirmations, as drafted by the International Swaps and Derivatives Association (ISDA), outline the details of each swap agreement. A representative of each party signs the confirmation, ensuring that they agree with all swap details (such as tenor and fixed/floating rates) and the steps taken in the event of default.

Comparative Advantage

AIM 29.5: Describe the comparative advantage argument for the existence of interest rate swaps and discuss some of the criticisms of this argument.

Let's return to companies X and Y and assume that they have access to borrowing for two years as specified in Figure 3.

Figure 3: Borrowing Rates for X and Y

Company	*Fixed Borrowing*	*Floating Borrowing*
Y	5.0%	LIBOR + 10 bps
X	6.5%	LIBOR + 100 bps

Company Y has an **absolute advantage** in both markets but a comparative advantage in the fixed market. Notice that the differential between X and Y in the fixed market is 1.5%, or 150 basis points (bps), and the corresponding differential in the floating market is only 90 basis points. When this is the case, Y has a comparative advantage in the fixed market, and X has a comparative advantage in the floating market. When a **comparative advantage** exists, a swap arrangement will reduce the costs of both parties. In this example, the net potential borrowing savings by entering into a swap is the difference between the differences, or 60 bps. In other words, by entering into a swap, the total savings shared between X and Y is 60 bps.

To better understand where the 60 bps comes from, suppose Y borrows fixed at 5% for two years, X borrows floating for two years at LIBOR + 1%, and then X and Y enter into a swap to transform their liabilities. Specifically, X pays Y fixed and Y pays X floating based on LIBOR. If we assume the net savings is split evenly, the net borrowing costs for X are then 6.2% and LIBOR – 20 bps for Y. Each has saved 30 bps for a total of 60 bps. If an intermediary were used, part of the 60 bps would be used to pay the bid-ask spread.

Problems With Comparative Advantage

A problem with the comparative advantage argument is that it assumes X can borrow at LIBOR + 1% over the life of the swap. It also ignores the credit risk taken on by Y by entering into the swap. If X were to raise funds by borrowing directly in the capital markets, no credit risk is taken, so perhaps the savings is compensation for that risk. The same criticisms exist when an intermediary is involved.

Valuing Interest Rate Swaps

The Discount Rate

AIM 29.6: Explain how the discount rates in a plain vanilla interest rate swap are computed.

Since a swap is nothing more than a sequence of cash flows, its value is determined by discounting each cash flow back to the valuation date. The question is, what is the appropriate *discount rate* to use? It turns out that the forward rates implied by either forward rate agreements (FRAs) or the convexity-adjusted Eurodollar futures are used to produce a LIBOR spot curve. The swap cash flows are then discounted using the corresponding spot rate from this curve. The following connection between forward rates and spot rates exists when continuous compounding is used:

$$R_{\text{forward}} = R_2 + (R_2 - R_1)\,\frac{T_1}{T_2 - T_1}$$

where:
R_i = spot rate corresponding with T_i years
R_{forward} = forward rate between T_1 and T_2

We will utilize this equation later when we value an interest rate swap using a sequence of forward rate agreements.

Valuing an Interest Rate Swap With Bonds

AIM 29.7: Value a plain vanilla interest rate swap based on two simultaneous bond positions.

Let's return to our two companies, X and Y, in our 2-year swap arrangement. From X's perspective, there are two series of cash flows—one fixed going out and one floating coming in. Essentially, X has a long position in a floating-rate note (since it is an inflow) and a short position in a fixed-rate note (since it is an outflow). From Y's perspective, it is exactly the opposite—Y has a short position in a floating-rate note (since it is an outflow) and a long position in a fixed-rate note (since it is an inflow).

If we denote the present value of the fixed-leg payments as B_{fix} and the present value of the floating-leg payments as B_{flt}, the value of the swap can be written for both X and Y as:

$$V_{swap}(X) = B_{flt} - B_{fix}$$

$$V_{swap}(Y) = B_{fix} - B_{flt}$$

Note that $V_{swap}(X) + V_{swap}(Y) = 0$. This is by design since the two positions are mirror images of one another. At inception of the swap, it is convention to select the fixed payment so that $V_{swap}(X) = V_{swap}(Y) = 0$. As expected floating rates in the future change, the swap value for each party is no longer zero.

Valuing an interest rate swap in terms of bond positions involves understanding that the value of a floating rate bond will be equal to the notional amount at any of its periodic settlement dates when the next payment is set to the market (floating) rate. Since $V_{swap} = \text{Bond}_{fixed} - \text{Bond}_{floating}$, we can value the fixed-rate bond using the spot rate curve and then discount the next (known) floating-rate payment plus the notional amount at the current discount rate. The following example illustrates this method.

Example: Valuing an interest rate swap

Consider a $1 million notional swap that pays a floating rate based on 6-month LIBOR and receives a 6% fixed rate semiannually. The swap has a remaining life of 15 months with pay dates at 3, 9, and 15 months. Spot LIBOR rates are as follows: 3 months at 5.4%; 9 months at 5.6%; and 15 months at 5.8%. The LIBOR at the last payment date was 5.0%. **Calculate** the value of the swap to the fixed-rate receiver using the bond methodology.

Answer:

$$B_{fixed} = \left(PMT_{fixed,\,3\text{ months}} \times e^{-(r \times t)}\right) + \left(PMT_{fixed,\,9\text{ months}} \times e^{-(r \times t)}\right) + \left[\left(\text{notional} + PMT_{fixed,\,15\text{ months}}\right) \times e^{-(r \times t)}\right]$$

$$B_{fixed} = \left(\$30{,}000 \times e^{-(0.054 \times 0.25)}\right) + \left(\$30{,}000 \times e^{-(0.056 \times 0.75)}\right) + \left[(\$1{,}000{,}000 + \$30{,}000) \times e^{-(0.058 \times 1.25)}\right]$$
$$= \$29{,}598 + \$28{,}766 + \$957{,}968 = \$1{,}016{,}332$$

$$B_{floating} = \left[\text{notional} + \left(\text{notional} \times \frac{r_{floating}}{2}\right)\right] \times e^{-(r \times t)}$$

$$= \left[\$1{,}000{,}000 + \left(\$1{,}000{,}000 \times \frac{0.05}{2}\right)\right] \times e^{-(0.054 \times 0.25)} = \$1{,}011{,}255$$

$$V_{swap} = \left(B_{fixed} - B_{floating}\right) = \$1{,}016{,}332 - \$1{,}011{,}255 = \$5{,}077$$

Figure 4 sums up the payments and present value factors.

Figure 4: Valuing an Interest Rate Swap With Two Bond Positions

Time	*Fixed Cash Flow*	*Floating Cash Flow*	*Present Value Factor*	*PV Fixed CF*	*PV Floating CF*
0.25 (3 months)	30,000	1,025,000	0.9866*	29,598	1,011,255
0.75 (9 months)	30,000		0.9589*	28,766	
1.25 (15 months)	1,030,000		0.9301*	957,968	
Total				1,016,332	1,011,255

* Note that some rounding has occurred.

Again we see that the value of the swap = 1,016,332 – 1,011,255 = $5,077.

Valuing an Interest Rate Swap With FRAs

AIM 29.8: Value a plain vanilla interest rate swap based on a sequence of forward rate agreements (FRAs).

At settlement, the payment made on a forward rate agreement is the notional amount multiplied by the difference between a market (floating) rate such as LIBOR and the contract (fixed) rate specified in the FRA. This is identical to a periodic payment on an interest rate swap when the reference floating rates and notional principal amounts are the same and the swap fixed rate is equal to the contract rate specified in the FRA. Viewed this way, we can see that an interest rate swap is equivalent to a series of FRAs. One way to value a swap would be to use expected forward rates to forecast the expected net cash flows and then discount these expected cash flows at the corresponding spot rates, consistent with forward rate expectations.

Example: Valuing an interest rate swap with FRAs

Consider the previous example on valuing an interest rate swap with two bond positions. An investor has a $1 million notional swap that pays a floating rate based on 6-month LIBOR and receives a 6% fixed rate semiannually. The swap has a remaining life of 15 months with pay dates at 3, 9, and 15 months. Spot LIBOR rates are as follows: 3 months at 5.4%; 9 months at 5.6%; and 15 months at 5.8%. The LIBOR at the last payment date was 5.0%. Calculate the value of the swap to the fixed-rate receiver using the FRA methodology.

Answer:

To calculate the value of the swap, we'll need to find the floating rate cash flows by calculating the expected forward rates via the LIBOR based spot curve.

The first floating rate cash flow is calculated in a similar fashion to the previous example.

LIBOR rate (last payment date): 5%.

Floating rate cash flow in 3 months: 1,000,000 × 0.05 / 2 = $25,000.

The second floating rate cash flow is calculated by finding the forward rate that corresponds to the period between 3 months and 9 months. To calculate forward rate for the period between 3 and 9 months, use the previously mentioned forward rate formula:

$$R_{\text{forward}} = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

$$R_{\text{forward}} = 0.056 + (0.056 - 0.054)\frac{0.25}{0.75 - 0.25} = 0.057 = 5.7\%$$

This rate is a continuously compounded rate, so we need to find the equivalent forward rate with semiannual compounding:

$R_{\text{forward (SC)}} = 2 \times [e^{(0.057/2)} - 1] = 0.05782 = 5.782\%$

Floating rate cash flow in 9 months: 1,000,000 × 0.05782 / 2 = $28,910

The third floating rate cash flow is calculated by finding the forward rate that corresponds to the period between 9 months and 15 months.

$$R_{\text{forward}} = 0.058 + (0.058 - 0.056)\frac{0.75}{1.25 - 0.75} = 0.061 = 6.1\%$$

$R_{\text{forward (SC)}} = 2 \times [e^{(0.061/2)} - 1] = 0.06194 = 6.1939\%$

Floating rate cash flow in 15 months: 1,000,000 × 0.061939 / 2 = $30,969

Figure 5: Valuing an Interest Rate Swap Based on a Sequence of FRAs

Time	*Fixed Cash Flow*	*Floating Cash Flow*	*Present Value Factor*	*PV Fixed CF*	*PV Floating CF*
0.25 (3 months)	30,000	25,000	0.9866*	29,598	24,665
0.75 (9 months)	30,000	28,910	0.9589*	28,766	27,721
1.25 (15 months)	30,000	30,969	0.9301*	27,902	28,803
Total				86,266	81,189

* Note that some rounding has occurred.

The value of the swap based on a sequence of FRAs = 86,266 – 81,189= $5,077.

As you can see from the previous two examples, valuing a swap based on a sequence of forward rate agreements produces the same result as valuing a swap based on two simultaneous bond positions.

Currency Swaps

AIM 29.9: Explain the mechanics of a currency swap and compute its cash flows.

AIM 29.12: Value a currency swap based on two simultaneous bond positions.

A **currency swap** exchanges both principal and interest rate payments with payments in different currencies. The exchange rate used in currency swaps is the spot exchange rate. The valuation and application of currency swaps is similar to the interest rate swap. However, since the principals in a currency swap are not the same currency, they are exchanged at the inception of the currency swap so that they have equal value using the spot exchange rate. Also, the periodic cash flows throughout the swap are not netted as they are in the interest rate swap.

Suppose we have two companies, A and B, that enter into a fixed-for-fixed currency swap with periodic payments annually. Company A pays 6% in Great Britain pounds (GBP) to Company B and receives 5% in U.S. dollars (USD) from Company B. Company A pays a principal amount to B of USD175 million, and B pays GBP100 million to A at the outset of the swap. Notice that A has effectively borrowed GBP from B and so it must pay interest on that loan. Similarly, B has borrowed USD from A. The cash flows in this swap are actually more easily computed than in an interest rate swap since both legs of the swap are fixed. Every period (12 months), A will pay GBP6 million to B, and B will pay USD8.75 million to A. At the end of the swap, the principal amounts are re-exchanged.

From Company A's perspective, there are two series of cash flows: one fixed GBP cash flow stream going out and one fixed USD cash flow stream coming in. Essentially, A has a long position in a USD-denominated note (since it's an inflow) and a short position in a GBP-denominated note (since it's an outflow).

If we denote the present value of the GBP-denominated payments as B_{GBP} and the present value of the USD payments as B_{USD}, the value of the swap in USD to Company A is:

$$V_{swap}(USD) = B_{USD} - (S_0 \times B_{GBP})$$

where:
S_0 = spot rate in USD per GBP

Example: Calculate the value of a currency swap

Suppose the yield curves in the United States and Great Britain are flat at 2% and 4%, respectively, and the current spot exchange rate is USD1.50 = GBP1. **Value** the currency swap just discussed assuming the swap will last for three more years.

Answer:

$$B_{USD} = 8.75e^{-0.02\times1} + 8.75e^{-0.02\times2} + 183.75e^{-0.02\times3} = \text{USD}190.03 \text{ million}$$

$$B_{GBP} = 6e^{-0.04\times1} + 6e^{-0.04\times2} + 106e^{-0.04\times3} = \text{GBP}105.32 \text{ million}$$

$$V_{swap} \text{ (to A in USD)} = 190.03 - (1.5 \times 105.32) = \text{USD}32.05 \text{ million}$$

AIM 29.13: Value a currency swap based on a sequence of FRAs.

The value of a currency swap can also be calculated based on a sequence of FRAs.

Example: Value of a currency swap with FRAs

Suppose the yield curves in the United States and Great Britain are flat at 2% and 4%, respectively, and the current spot exchange rate is USD1.50 = GBP1.

Compute the value of the currency swap discussed previously using a sequence of FRAs to Company A. Assume the swap will last for three more years.

The corresponding forward rates are as follows:

Figure 6: Forward Rates

Year 1	$1.47/£
Year 2	$1.44/£
Year 3	$1.41/£

Professor's Note: The year 1 forward rate is calculated as follows: $F_1 = 1.5e^{(0.02-0.04)\times1} = \$1.47/£$. *Interest rate parity suggests that the dollar will appreciate relative to the pound, so the $/£ forward rate will decline (i.e., it will take fewer USD to buy 1 GBP). We will discuss interest rate parity in the foreign exchange risk topic (Topic 34).*

Answer:

Figure 7 denotes the cash flows and forward rates for this currency swap.

Figure 7: Valuing a Currency Swap Based on a Sequence of FRAs

Time	*USD Cash Flow*	*GBP Cash Flow*	*Forward Rate*	*$ Value of £*	*Net Cash Flows*	*PV of Net CF*
1	8.75	6	1.47	8.82	–0.07	–0.069
2	8.75	6	1.44	8.64	0.11	0.106
3	8.75	6	1.41	8.46	0.29	0.273
3	175	100	1.41	141	34	32.02
Total						32.33*

* Note some rounding has occurred.

Ignoring the rounding differences, we see that the value of the currency swap to Company A is 32 million using both the two simultaneous bond positions and the forward rate agreements.

Using a Currency Swap to Transform Existing Positions

AIM 29.11: Explain how a currency swap can be used to transform an asset or liability and calculate the resulting cash flows.

Currency swaps can be combined with existing positions to completely alter the risk of a liability or an asset. For example, suppose that Company A has a dollar-based liability. By entering into a currency swap, the liability has become a pound-based liability at the GBP fixed (or floating) rate.

Comparative Advantage

AIM 29.10: Describe the comparative advantage argument for the existence of currency swaps.

Comparative advantage is also used to explain the success of currency swaps. Typically, a domestic borrower will have an easier time borrowing in his own currency. This often results in comparative advantages that can be exploited by using a currency swap. The argument is directly analogous to that used for interest rate swaps. Suppose A and B have the 5-year borrowing rates in the United States and Germany (EUR) shown in Figure 8.

Figure 8: Borrowing Rates

Borrowing Rates for A and B		
Company	*USD Borrowing*	*EUR Borrowing*
A	5.0%	7.0%
B	6.0%	7.5%

Company A needs EUR, and Company B needs USD. Company A has an absolute advantage in both markets but a comparative advantage in the USD market. Notice that the differential between A and B in the USD market is 1%, or 100 basis points (bps), and the corresponding differential in the EUR market is only 50 basis points. When this is the case, A has a comparative advantage in the USD market, and B has a comparative advantage in the EUR market. The net potential borrowing savings by entering into a swap is the difference between the differences, or 50 bps. In other words, by entering into a currency swap, the savings for both A and B totals 50 bps.

Swap Credit Risk

AIM 29.14: Discuss the role of credit risk inherent in an existing swap position.

Because $V_{swap}(A) + V_{swap}(B) = 0$, whenever one side of a swap has a positive value, the other side must be negative. For example, if $V_{swap}(A) > 0$, $V_{swap}(B) < 0$. As $V_{swap}(A)$ increases in value, $V_{swap}(B)$ must become more negative. This results in increased credit risk to A since the likelihood of default increases as B has larger and larger payments to make to A. However, the potential losses in swaps are generally much smaller than the potential losses from defaults on debt with the same principal. This is because the value of swaps is generally much smaller than the value of the debt.

Other Types of Swaps

AIM 29.15: List and define other types of swaps, including commodity, volatility and exotic swaps.

In an **equity swap**, the return on a stock, a portfolio, or a stock index is paid each period by one party in return for a fixed-rate or floating-rate payment. The return can be the capital appreciation or the total return including dividends on the stock, portfolio, or index.

In order to reduce equity risk, a portfolio manager might enter into a 1-year quarterly pay S&P 500 index swap and agree to receive a fixed rate. The percentage increase in the index each quarter is netted against the fixed rate to determine the payment to be made. If the index return is negative, the fixed-rate payer must also pay the percentage decline in the index to the portfolio manager. Uniquely among swaps, equity swap payments can be floating on both sides and the payments are not known until the end of the quarter. With interest rate swaps, both the fixed and floating payments are known at the beginning of the period for which they will be paid.

A swap on a single stock can be motivated by a desire to protect the value of a position over the period of the swap. To protect a large capital gain in a single stock, and to avoid a sale for tax or control reasons, an investor could enter into an equity swap as the equity-returns payer and receive a fixed rate in return. Any decline in the stock price would be paid to the investor at the settlement dates, plus the fixed-rate payment. If the stock appreciates, the investor must pay the appreciation less the fixed payment.

A **swaption** is an option which gives the holder the right to enter into an interest rate swap. Swaptions can be American- or European-style options. Like any option, a swaption is purchased for a premium that depends on the strike rate (the fixed rate) specified in the swaption.

Firms may enter into **commodity swap** agreements where they agree to pay a fixed rate for the multi-period delivery of a commodity and receive a corresponding floating rate based on the average commodity spot rates at the time of delivery. Although many commodity swaps exist, the most common use is to manage the costs of purchasing energy resources such as oil and electricity.

A **volatility swap** involves the exchanging of volatility based on a notional principal. One side of the swap pays based on a pre-specified volatility while the other side pays based on historical volatility.

As you can see, many different types of swaps exist. Some additional examples include: accrual swaps, cancelable swaps, index amortizing rate swaps, and constant maturity swaps. Swaps are also sometimes created for exotic structures. An example of an **exotic swap** was between Procter and Gamble and Banker's Trust where P&G's payments were based on the commercial paper rate.

Key Concepts

1. A plain vanilla interest rate swap exchanges floating-rate payments (LIBOR) for fixed-rate payments over the life of the swap.
2. The floating rate payments at *t* in a plain vanilla interest rate swap are computed using the floating rate at time t – 1.
3. Swaps can be combined with existing asset and liability positions to drastically change the interest rate risk.
4. A swap dealer or financial intermediary facilitates the ability to enter into swaps.
5. The comparative advantage argument suggests that when one of two borrowers has a comparative advantage in either the fixed- or floating-rate market, both borrowers will be better off by entering into a swap to exploit the advantage.
6. The comparative advantage argument is flawed in that it assumes rates can be borrowed for the life of the swap. It also ignores the credit risk associated with the swap that does not exist if funds were raised directly in the capital markets.
7. A swap is equivalent to a simultaneous position in two bonds or a series of forward rate agreements (FRAs).
8. The value of a swap to the fixed-rate receiver at a point in time is the difference between the present value of the remaining fixed-rate payments and the present value of the remaining floating-rate payments.
9. A currency swap exchanges interest rate payments in two different currencies.
10. Credit risk is an important factor in existing swap positions, although potential losses are usually smaller than that with debt agreements.
11. Many different types of swaps exist. Examples of swaps, in addition to interest rate swaps and commodity swaps, include: equity swaps, commodity swaps, and volatility swaps.

Concept Checkers

Use the following data to answer Question 1.

Two companies, C and D, have the borrowing rates shown in the following table.

Borrowing Rates for C and D

Company	*Fixed Borrowing*	*Floating Borrowing*
C	10%	LIBOR + 50 bps
D	12%	LIBOR + 100 bps

1. According to the comparative advantage argument, what is the total potential savings for C and D if they enter into an interest rate swap?
   A. 0.5%.
   B. 1.0%.
   C. 1.5%.
   D. 2.0%.

2. Which of the following is most accurate regarding the credit risk of a currency swap? As the value of the:
   I. domestic currency leg increases, so does the credit risk of the domestic currency payer.
   II. foreign currency leg increases, so does the credit risk of the foreign currency payer.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

3. Which of the following would properly transform a floating-rate liability to a fixed-rate liability? Enter into a pay:
   A. foreign currency swap.
   B. fixed interest rate swap.
   C. domestic currency swap.
   D. floating interest rate swap.

4. Use the following information to determine the value of the swap to the floating rate payer using the bond methodology. Assume we are at the floating rate reset date.
   - $1 million notional value, semiannual, 18-month maturity.
   - Spot LIBOR rates: 6 months, 2.6%; 12 months, 2.65%; 18 months, 2.75%.
   - The fixed rate is 2.8%, with semiannual payments.

   A. –$66.
   B. $476.
   C. $3,425.
   D. $5,077.

5. Suppose Company X pays 5% annually (in euros) to Company Y and receives 4% annually (in dollars). Company X pays a principal amount of $150 million to Y, and Y pays a €100 million to X at the inception of the swap. Assume the yield curve is flat in the United States and in Germany (Europe). The U.S. rate is 3%, and the German rate is 5%. The current spot exchange rate is $1.45/€. What is the value of the currency swap to Company X using the bond methodology if it is expected to last for two more years?
   A. $3.53 million.
   B. $52.98 million.
   C. $8.09 million.
   D. $12.74 million.

Concept Checker Answers

1. C The difference of the differences is (12% – 10%) – [LIBOR + 1% – (LIBOR + 0.5%)] = 1.5%.

2. D As one currency (A) appreciates relative to another currency (B), the value of a currency swap increases on behalf of the currency A payer. As a result, the credit risk of the currency B payer increases.

3. B The fixed interest rate swap will allow for the conversion of a floating-rate liability to a fixed-rate liability.

4. B $B_{fix} = [\$14{,}000 \times e^{-(0.026\times0.5)}] + [\$14{,}000 \times e^{-(0.0265\times1.0)}] + [(\$1{,}000{,}000 + \$14{,}000) \times e^{-(0.0275\times1.5)}] = \$13{,}819 + \$13{,}634 + \$973{,}023 = \$1{,}000{,}476$

   Note that we are at a (semiannual) reset date, so the floating rate portion has a value equal to the notional amount.

   $V_{swap} = (B_{fix} - B_{floating}) = \$1{,}000{,}476 - \$1{,}000{,}000 = \$476$

5. C $B_{\$} = 6e^{-0.03\times1} + 156e^{-0.03\times2} = \$5.82 + \$146.92 = \$152.74$

   $B_{€} = 5e^{-0.05\times1} + 105e^{-0.05\times2} = €4.76 + €95.00 = €99.76$

   V_{swap} (to X) = 152.74 – (1.45 × 99.76) = \$8.09 million

The following is a review of the Financial Markets and Products principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Properties of Stock Options

Topic 30

Exam Focus

Stock options have several properties relating both to their value and to the factors that affect their price. Six factors affect option prices: the current value of the stock; the strike price; the time to expiration; the volatility of the stock price; the risk-free rate; and dividends. The value of stock options have upper and lower pricing bounds. Be familiar with these pricing bounds as well as the relationships that exist between the value of European and American options.

Six Factors That Affect Option Prices

AIM 30.1: Identify the six factors that affect an option's price and discuss how these six factors affect the price for both European and American options.

The following six factors will impact the value of an option:

1. S_0 = current stock price.
2. X = strike price of the option.
3. T = time to expiration of the option.
4. r = short-term risk-free interest rate over *T*.
5. D = present value of the dividend of the underlying stock.
6. σ = expected volatility of stock prices over *T*.

When evaluating a change in any one of the factors, hold the other factors constant.

Current Price of the Stock

For call options, as *S* increases (decreases), the value of the call increases (decreases). For put options, as *S* increases (decreases), the value of the put decreases (increases). This simply states that as an option becomes closer to or more in-the-money, its value increases.

Strike Price of the Option

The effect of strike prices on option values will be exactly the opposite of the effect of the current price of the stock. For call options, as *X* increases (decreases), the value of the call decreases (increases). For put options, as *X* increases (decreases), the value of the put increases (decreases). This is the same as the logic for the current price of the stock: the option's value will increase as it becomes closer to or more in-the-money.

The Time to Expiration

For American-style options, increasing time to expiration will increase the option value. With more time, the likelihood of being in-the-money increases. A general statement cannot be made for European-style options. Suppose we have a 1-month and 3-month call option on the same underlying with the same exercise price. Also suppose a large dividend is expected to be paid in two months. Because the stock price and 3-month option price will fall when the dividend is paid in two months, the 1-month option may be worth more than the 3-month option.

The Risk-Free Rate Over the Life of the Option

As the risk-free rate increases, the value of the call (put) will increase (decrease). The intuition behind this property involves arbitrage arguments that require the use of synthetic securities.

Dividends

The option owner does not have access to the cash flows of the underlying stock, and the stock price decreases when a dividend is paid. Thus, as the dividend increases, the value of the call (put) will decrease (increase).

Volatility of the Stock Price Over the Life of the Option

Volatility is the friend of all options. As volatility increases, option values increase. This is due to the asymmetric payoff of options. Since long option positions have a maximum loss equal to the premium paid, increased volatility only increases the chances that the option will expire in-the-money. Many consider volatility to be the most important factor for option valuation.

Figure 1 summarizes the factors' effects on option prices: "+" indicates a positive effect on option price from an increase in the factor, and "–" is a negative effect on option price.

Figure 1: Summary of Effects of Increasing a Factor on the Price of an Option

Factor	*European Call*	*European Put*	*American Call*	*American Put*
S	+	–	+	–
X	–	+	–	+
T	?	?	+	+
σ	+	+	+	+
r	+	–	+	–
D	–	+	–	+

Upper and Lower Pricing Bounds

AIM 30.2: Identify, interpret and compute upper and lower bounds for option prices.

In addition to those previously introduced, consider the following variables:

- c = value of a European call option.
- C = value of an American call option.
- p = value of a European put option.
- P = value of an American put option.
- S_T = value of the stock at expiration.

Also, assume in the following examples that there are no transaction costs, all profits are taxed at the same rate, and borrowing and lending can be done at the risk-free rate.

Upper Pricing Bounds for European and American Options

A call option gives the right to purchase one share of stock at a certain price. Under no circumstance can the option be worth more than the stock. If it were, everyone would sell the option and buy the stock and realize an arbitrage profit. We express this as:

$$c \leq S_0 \text{ and } C \leq S_0$$

Similarly, a put option gives the right to sell one share of stock at a certain price. Under no circumstance can the put be worth more than the sale or strike price. If it were, everyone would sell the option and invest the proceeds at the risk-free rate over the life of the option. We express this as:

$$p \leq X \text{ and } P \leq X$$

For a European put option, we can further reduce the upper bound. Since it cannot be exercised early, it can never be worth more than the present value of the strike price:

$$p \leq Xe^{-rT}$$

Lower Pricing Bounds for European Calls on Nondividend-Paying Stocks

Consider the following two portfolios:

- Portfolio P_1: one European call, *c*, with exercise price *X* plus a zero-coupon risk-free bond that pays *X* at *T*.
- Portfolio P_2: one share of the underlying stock, *S*.

At expiration, *T*, Portfolio P_1 will always be the greater of *X* (when the option expires out-of-the-money) or S_T (when the option expires in-the-money). Portfolio P_2, on the other hand, will always be worth S_T. Therefore, P_1 is always worth at least as much as P_2 at

expiration. If we know that at T, $P_1 \geq P_2$, then it always has to be true because if it were not, arbitrage would be possible. Therefore, we can state the following:

$$c + Xe^{-rT} \geq S_0$$

Since the value of a call option cannot be negative (if the option expires out-of-the-money, its value will be zero), the lower bound for a European call on a nondividend-paying stock is:

$$c \geq \max(S_0 - Xe^{-rT}, 0)$$

Lower Pricing Bounds for European Puts on Nondividend-Paying Stocks

Consider the following two portfolios:

- Portfolio P_3: one European put, p, plus one share of the underlying stock, S.
- Portfolio P_4: zero-coupon risk-free bond that pays X at T.

At expiration, T, Portfolio P_3 will always be the greater of X (when the option expires in-the-money) or S_T (when the option expires out-of-the-money). Portfolio P_4, on the other hand, will always be worth X. Therefore, P_3 is always worth at least as much as P_4 at expiration. If we know that at T, $P_3 \geq P_4$, it has to be true always because if it were not, arbitrage would be possible. Therefore, we can state the following:

$$p + S_0 \geq Xe^{-rT}$$

Since the value of a put option cannot be negative (if the option expires out-of-the-money, its value will be zero), the lower bound for a European put on a nondividend-paying stock is:

$$p \geq \max(Xe^{-rT} - S_0, 0)$$

Computing Option Values Using Put-Call Parity

AIM 30.3: Explain put-call parity and calculate, using the put-call parity on a non-dividend-paying stock, the value of a European and American option, respectively.

The derivation of **put-call parity** is based on the payoffs of two portfolio combinations, a fiduciary call and a protective put.

A *fiduciary call* is a combination of a pure-discount (i.e., zero coupon), riskless bond that pays X at maturity and a call with exercise price X. The payoff for a fiduciary call at expiration is X when the call is out of the money, and $X + (S - X) = S$ when the call is in the money.

A *protective put* is a share of stock together with a put option on the stock. The expiration date payoff for a protective put is $(X - S) + S = X$ when the put is in the money, and S when the put is out of the money.

Professor's Note: When working with put-call parity, it is important to note that the exercise prices on the put and the call and the face value of the riskless bond are all equal to X.

When the put is in the money, the call is out of the money, and both portfolios pay X at expiration.

Similarly, when the put is out of the money and the call is in the money, both portfolios pay S at expiration.

Put-call parity holds that portfolios with identical payoffs must sell for the same price to prevent arbitrage. We can express the put-call parity relationship as:

$$c + Xe^{-rT} = S + p$$

Equivalencies for each of the individual securities in the put-call parity relationship can be expressed as:

$$\begin{aligned} S &= c - p + Xe^{-rT} \\ p &= c - S + Xe^{-rT} \\ c &= S + p - Xe^{-rT} \\ Xe^{-rT} &= S + p - c \end{aligned}$$

The single securities on the left-hand side of the equations all have exactly the same payoffs as the portfolios on the right-hand side. The portfolios on the right-hand side are the "synthetic" equivalents of the securities on the left. Note that the options must be European-style and the puts and calls must have the same exercise price for these relations to hold.

For example, to synthetically produce the payoff for a long position in a share of stock, you use the relationship:

$$S = c - p + Xe^{-rT}$$

This means that the payoff on a long stock can be synthetically created with a long call, a short put, and a long position in a risk-free discount bond.

The other securities in the put-call parity relationship can be constructed in a similar manner.

Professor's Note: After expressing the put-call parity relationship in terms of the security you want to synthetically create, the sign on the individual securities will indicate whether you need a long position (+ sign) or a short position (– sign) in the respective securities.

Example: Call option valuation using put-call parity

Suppose that the current stock price is \$52 and the risk-free rate is 5%. You have found a quote for a 3-month put option with an exercise price of \$50. The put price is \$1.50, but due to light trading in the call options, there was not a listed quote for the 3-month, \$50 call. Estimate the price of the 3-month call option.

Answer:

Rearranging put-call parity, we find that the call price is:

$$\text{call} = \text{put} + \text{stock} - Xe^{-rT}$$

$$\text{call} = \$1.50 + \$52 - \$50e^{-0.0125} = \$4.12$$

This means that if a 3-month, \$50 call is available, it should be priced at \$4.12 per share.

Lower Pricing Bounds for an American Call Option on a Nondividend-Paying Stock

AIM 30.4: Explain the early exercise features of American call and put options on a non-dividend-paying stock and the price effect early exercise may have.

Recall the following equation from our earlier discussion of the lower pricing bounds for a *European* call option:

$$c \geq \max(S_0 - Xe^{-rT}, 0)$$

Since the only difference between an American option and a European option is that the American option can be exercised early, American options can always be used to replicate their corresponding European options simply by choosing not to exercise them until expiration. Therefore, it follows that:

$$C \geq c \geq \max(S_0 - Xe^{-rT}, 0)$$

Note that when an American call is exercised, it is only worth $S_0 - X$. Since this value is never larger than $S_0 - Xe^{-rT}$ for any r and $T > 0$, it is never optimal to exercise early. In other words, the investor can keep the cash equal to X, which would be used to exercise the option early, and invest that cash to earn interest until expiration. Since exercising the American call early means that the investor would have to forgo this interest, it is never optimal to exercise an American call on a nondividend-paying stock before the expiration date (i.e., c = C).

Lower Pricing Bounds for an American Put Option on a Nondividend-Paying Stock

While it is never optimal to exercise an American call on a nondividend-paying stock, American puts are optimally exercised early if they are sufficiently in-the-money. If an option is sufficiently in-the-money, it can be exercised, and the payoff $(X - S_0)$ can be invested to earn interest. In the extreme case when S_0 is close to zero, the future value of the exercised cash value, Xe^{rT}, is always worth more than a later exercise, *X*. We know that:

$P \geq p \geq \max(Xe^{-rT} - S_0, 0)$ for the same reasons that $C \geq c$

However, we can place an even stronger bound on an American put since it can always be exercised early:

$$P \geq \max(X - S_0, 0)$$

Figure 2 summarizes what we now know regarding the boundary prices for American and European options.

Figure 2: Lower and Upper Bounds for Options

Option	*Minimum Value*	*Maximum Value*
European call	$c \geq \max(0, S_0 - Xe^{-rT})$	S_0
American call	$C \geq \max(0, S_0 - Xe^{-rT})$	S_0
European put	$p \geq \max(0, Xe^{-rT} - S_0)$	Xe^{-rT}
American put	$P \geq \max(0, X - S_0)$	X

Professor's Note: For the exam, know the price limits in Figure 2. You will not be asked to derive them, but you may be expected to use them.

Example: Minimum prices for American vs. European puts

Compute the lowest possible price for 4-month American and European 65 puts on a stock that is trading at 63 when the risk-free rate is 5%.

Answer:

$$P \geq \max(0, X - S_0) = \max(0, 2) = \$2$$

$$p \geq \max(0, Xe^{-rT} - S_0) = \max(0, 65e^{-0.0167} - 63) = \$0.92$$

Example: Minimum prices for American vs. European calls

Compute the lowest possible price for 3-month American and European 65 calls on a stock that is trading at 68 when the risk-free rate is 5%.

Answer:

$C \geq \max(0, S_0 - Xe^{-rT}) = \max(0, 68 - 65e^{-0.0125}) = \$3.81$

$c \geq \max(0, S_0 - Xe^{-rT}) = \max(0, 68 - 65e^{-0.0125}) = \$3.81$

Relationship Between American Call Options and Put Options

Put-call parity only holds for European options. For American options, we have an inequality. This inequality places upper and lower bounds on the difference between the American call and put options.

$$S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$$

Example: American put option bounds

Consider an American call and put option on stock XYZ. Both options have the same 1-year expiration and a strike price of \$20. The stock is currently priced at \$22, and the annual interest rate is 6%. What are the upper and lower bounds on the American put option if the American call option is priced at \$4?

Answer:

The upper and lower bounds on the difference between the American call and American put options are:

$S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$

$S_0 - X = 22 - 20 = \$2$

$S_0 - Xe^{-rT} = 22 - 20e^{-0.06(1)} = 22 - 18.84 = \$3.16$

$\$2 \leq C - P \leq \$3.16$

or

$-\$2 \geq P - C \geq -\$3.16$

Therefore, when the American call is valued at \$4, the upper and lower bounds on the American put option will be:

$\$2 \geq P \geq \$0.84$

The Impact of Dividends on Option Pricing Bounds

AIM 30.5: Discuss the effects dividends have on the put-call parity, the bounds of put and call option prices, and on the early exercise feature of American options.

Since most stock options have an expiration of less than a year, dividends can be estimated fairly accurately. Recall that to prevent arbitrage, when a stock pays a dividend, its value must decrease by the amount of the dividend. This increases the value of a put option and decreases the value of a call option.

Consider the following portfolios:

- Portfolio P_6: one European call option, *c*, plus cash equal to $D + Xe^{-rT}$.
- Portfolio P_7: one share of the underlying stock, *S*.

Similar to the development of the $c \geq \max(S_0 - Xe^{-rT}, 0)$ equation, Portfolio P_6 is always at least as large as P_7, or:

$$c \geq S_0 - D - Xe^{-rT}$$

All else equal, the payment of a dividend will reduce the lower pricing bound for a call option.

For put options:

- Portfolio P_8: one European put, *p*, plus one share of the underlying stock, *S*.
- Portfolio P_9: cash equal to $D + Xe^{-rT}$.

Using the same development as the $p \geq \max(Xe^{-rT} - S_0, 0)$ equation:

$$p \geq D + Xe^{-rT} - S_0$$

All else equal, the payment of a dividend will increase the lower pricing bound for a put option.

Impact of Dividends on Early Exercise for American Calls and Put-Call Parity

When the dividend is large enough, American calls might be optimally exercised early. This will be the case if the amount of the dividend exceeds the amount of interest that is forgone as a result of the early exercise. Note that if a large dividend makes early exercise optimal, exercise should take place immediately before the ex-dividend date. Put-call parity is adjusted for dividends in the following manner:

$$p + S_0 = c + D + Xe^{-rT}$$

This equation is verified using the same development as was used to derive the $p + S_0 = c + Xe^{-rT}$ equation. The $S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$ equation that we used to show the relationship between American call and put options is modified as follows:

$$S_0 - X - D \leq C - P \leq S_0 - Xe^{-rT}$$

Key Concepts

1. Six factors influence the value of an option: current value of the underlying asset (stock); the strike price; the time to expiration of the option; the volatility of the stock price; the risk-free rate; and dividends.
2. With the exception of time to expiration, all of the above factors affect European- and American-style options in the same way.
3. Call options cannot be worth more than the underlying security, and put options cannot be worth more than the strike price.
4. When the stock does not pay a dividend, European call options cannot be worth less than the difference between the current stock price and the present value of the strike price. European put options cannot be worth less than the difference between the present value of the strike price and the current stock price.
5. Put-call parity is a no-arbitrage relationship for European-style options with the same characteristics. It states that a portfolio consisting of a call option and a zero-coupon bond with a face value equal to the strike must have the same value as a portfolio consisting of the corresponding put option and the stock:

   $$p + S_0 = c + Xe^{-rT}$$
6. It is never optimal to exercise an American call option on nondividend-paying stock prior to expiration.
7. American put options on nondividend-paying stocks can be optimally exercised prior to expiration if the put is sufficiently in-the-money.
8. Call options are always worth more than corresponding put options prior to expiration when both are at-the-money.
9. The difference between prices of an American call and corresponding put is bounded below by the difference between the current stock price and strike price, and above by the difference between the current stock price and the present value of the strike price.
10. Dividends will affect the pricing bounds and parity relationships for both calls and puts.

Concept Checkers

1. Which of the following will not cause a decrease in the value of a European call option position on XYZ stock?
   A. XYZ declares a 3-for-1 stock split.
   B. XYZ raises its quarterly dividend from $0.15 per share to $0.17 per share.
   C. The Federal Reserve lowers interest rates by 0.25% in an effort to stimulate the economy.
   D. Investors believe the volatility of XYZ stock has declined.

2. Consider a European put option on a stock trading at $50. The put option has an expiration of six months, a strike price of $40, and a risk-free rate of 5%. The lower bound and upper bound on the put are:
   A. $10, $40.00.
   B. $10, $39.01.
   C. $0, $40.00.
   D. $0, $39.01.

3. Consider a 1-year European put option that is currently valued at $5 on a $25 stock and a strike of $27.50. The 1-year risk-free rate is 6%. Which of the following is closest to the value of the corresponding call option?
   A. $0.00.
   B. $3.89.
   C. $4.10.
   D. $5.00.

4. Consider an American call and put option on the same stock. Both options have the same 1-year expiration and a strike price of $45. The stock is currently priced at $50, and the annual interest rate is 10%. Which of the following could be the difference in the two option values?
   A. $4.95.
   B. $7.95.
   C. $9.35.
   D. $12.50.

5. According to put-call parity for European options, purchasing a put option on ABC stock would be equivalent to:
   A. buying a call, buying ABC stock, and buying a zero-coupon bond.
   B. buying a call, selling ABC stock, and buying a zero-coupon bond.
   C. selling a call, selling ABC stock, and buying a zero-coupon bond.
   D. buying a call, selling ABC stock, and selling a zero-coupon bond.

Concept Checker Answers

1. **A** After a stock split, both the price of the stock and the strike price of the option will be adjusted, so the value of the option position will be the same. An increase in the dividend, a lower risk-free interest rate, and lower volatility of the price of the underlying stock, will all decrease the value of a European call option.

2. **D** The upper bound is the present value of the exercise price: $\$40 \times e^{-0.05\times0.5} = \$39.01$. Since the put is out-of-the-money, the lower bound is zero.

3. **C** $c = p - Xe^{-rT} + S_0 = \$5 - \$27.50e^{-0.06\times1} + \$25 = \$4.10$

4. **B** The upper and lower bounds are: $S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$ or $\$5 \leq C - P \leq \$9.28$. Only \$7.95 falls within the bounds.

5. **B** The formula for put-call parity is $p + S_0 = c + Xe^{-rT}$. Rearranging to solve for the price of a put, we have $p = c - S_0 + Xe^{-rT}$.

The following is a review of the Financial Markets and Products principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Trading Strategies Involving Options

Topic 31

Exam Focus

Traders and investors use option-based trading strategies to create an extraordinary spectrum of payoff profiles. This enables investors to take positions based on almost any possible expectation of the underlying stock over the life of the options. This topic describes the common option trading strategies and implementation. For the exam, know the general payoff graphs for each strategy discussed. In addition, know how to calculate the payoff for some of the more popular strategies including protective put, covered call, bull call spread, butterfly spread, and straddle.

Basics of Put Options and Call Options

Option contracts have asymmetric payoffs. The buyer of an option has the right to exercise the option but is not obligated to exercise. Therefore, the maximum loss for the buyer of an option contract is the loss of the price (premium) paid to acquire the position, while the potential gains in some cases are theoretically infinite. Because option contracts are a zero-sum game, the seller of the option contract could incur substantial losses, but the maximum potential gain is the amount of the premium received for writing the option.

To understand the potential returns, we need to introduce the standard symbols used to represent the relevant factors:

- X = strike price or exercise price specified in the option contract (a fixed value)
- S_t = price of the underlying asset at time t
- C_t = the market value of a call option at time t
- P_t = the market value of a put option at time t
- t = the time subscript, which can take any value between 0 and T, where T is the maturity or expiration date of the option

Call Options

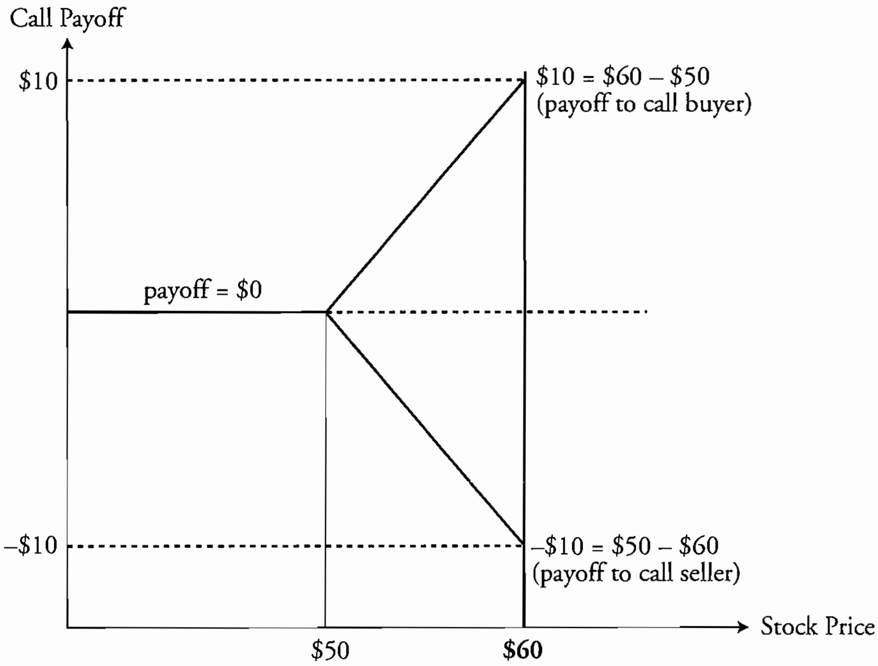
A *call option* gives the *owner* the right, but not the obligation, to buy the stock from the seller of the option. The owner is also called the *buyer* or the holder of the *long position*. The buyer benefits, at the expense of the option *seller*, if the underlying stock price is greater than the exercise price. The option *seller* is also called the *writer* or holder of the *short position*.

At maturity, time *T*, if the price of the underlying stock is less than or equal to the strike price of a call option (i.e., $S_T \leq X$), the payoff is zero, so the option owner would not exercise the option. On the other hand, if the stock price is higher than the exercise price (i.e., $S_T > X$) at maturity, then the payoff of the call option is equal to the difference between the market price and the strike price ($S_T - X$). The "payoff" (at the option's maturity) to the call option seller, which will be at most zero, is the mirror image (opposite sign) of the payoff to the buyer.

Because of the linear relationships between the value of the option and the price of the underlying asset, simple graphs can clearly illustrate the possible value of option contracts at the expiration date. Figure 1 illustrates the payoff of a call with an exercise price equal to 50.

Professor's Note: An option payoff graph ignores the initial cost of the option.

Figure 1: Payoff of Call With Exercise Price Equal to 50



Example: Payoff to the writer of a call option

An investor writes an at-the-money call option on a stock with an exercise price of 50 (X = 50). If the stock price rises to $60, what will be the *payoff* to the owner and seller of the call option?

Answer:

The call option may be exercised with the holder of the long position buying the stock from the writer at 50 for a $10 gain. The payoff to the option buyer is $10, and the payoff to the option writer is *negative* $10. This is illustrated in Figure 1, and as mentioned, does not include the premium paid for the option.

This example shows just how easy it is to determine option payoffs. At expiration time *T* (the option's maturity), the payoff to the option owner, represented by C_T, is:

$C_T = S_T - X$ if $S_T > X$

$C_T = 0$ if $S_T \leq X$

Another popular way of writing this is with the "max (0, variable)" notation, as you have seen in the previous topic. If the variable in this expression is greater than zero, then max (0, variable) = variable; if the variable's value is less than zero, then max (0, variable) = 0. Thus, letting the variable be the quantity $S_0 - X$, we can write:

$$C_T = \max(0, S_T - X)$$

The payoff to the option seller is the negative value of these numbers. In what follows, we will always talk about payoff in terms of the option owner unless otherwise stated. We should note that max $(0, S_t - X)$, where $0 < t < T$, is also the payoff if the owner decides to exercise the call option early. In this topic, we will only consider time T in our analysis.

Although our focus here is not to calculate C_t, we should clearly define it as the initial cost of the call when the investor purchases at time 0, which is T units of time before T. C_0 is also called the **premium**. Thus, we can write that the profit to the owner at t = T is:

$$\text{profit} = C_T - C_0$$

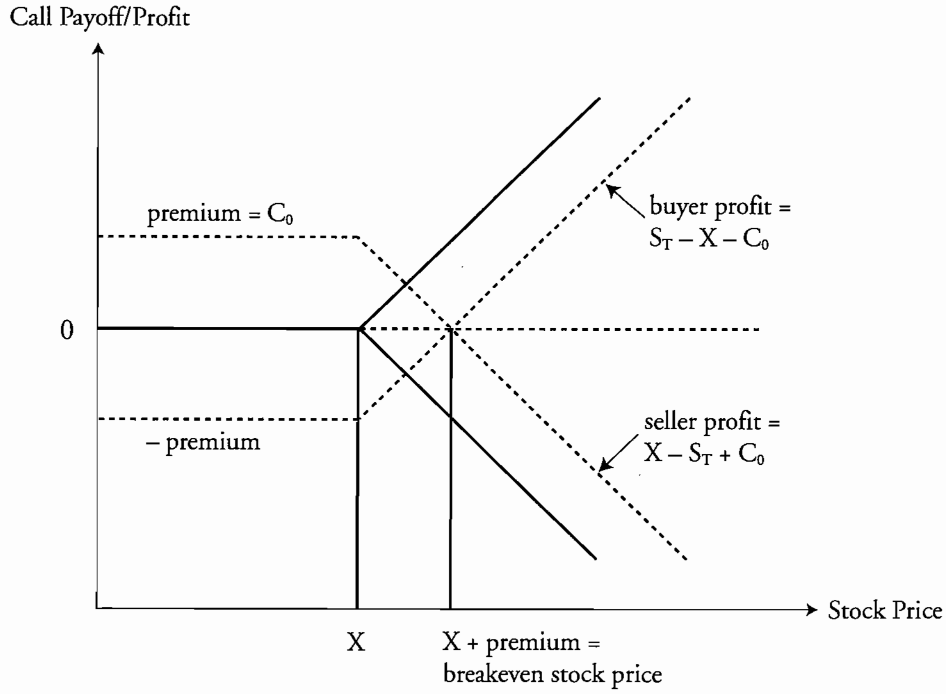
This says that at time T the owner's profit is the option payoff minus the premium paid at time 0. Incorporating C_0 into Figure 1 gives us the profit diagram for a call at expiration, and this is Figure 2.

Figure 2 illustrates an important point, which is that the profit to the owner is negative when the stock price is less than the exercise price plus the premium. At expiration, we can say that:

if $S_T < X + C_0$ then: call buyer profit < 0 < call seller profit
if $S_T = X + C_0$ then: call buyer profit = 0 = call seller profit
if $S_T > X + C_0$ then: call buyer profit > 0 > call seller profit

The **breakeven price** is a very descriptive term that we use for $X + C_0$, or X + premium.

Figure 2: Profit Diagram for a Call at Expiration



Put Options

If you understand the properties of a call, the properties of a put should come to you fairly easily. A put option gives the owner the right to sell a stock to the seller of the put at a specific price. At expiration, the buyer benefits if the price of the underlying is less than the exercise price X:

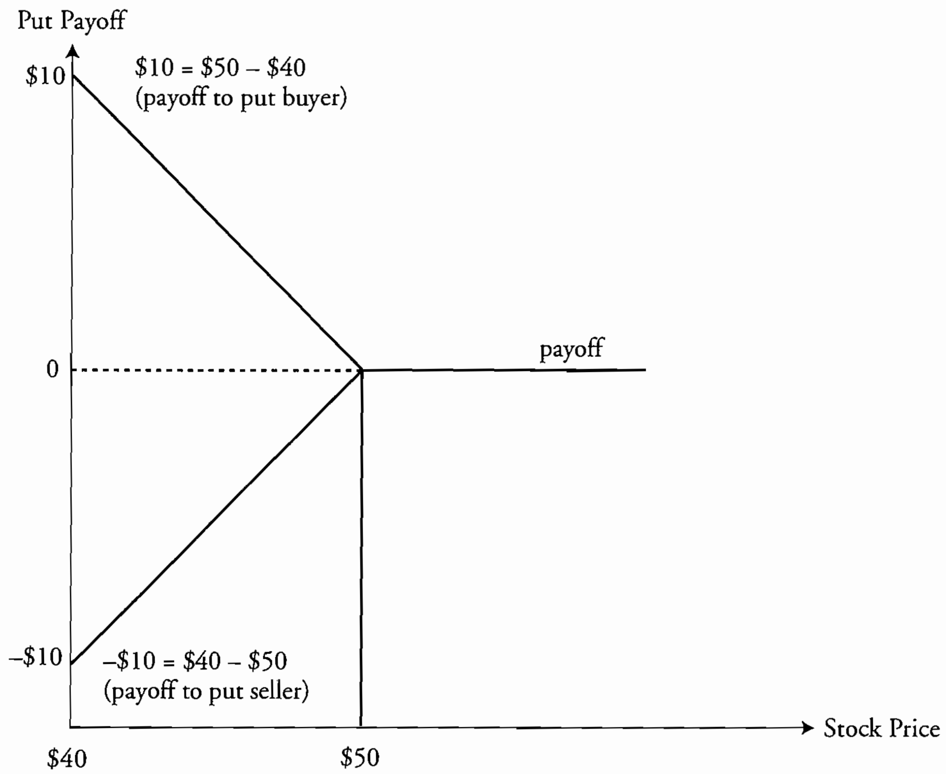
$$P_T = X - S_T \text{ if } S_T < X$$
$$P_T = 0 \quad \text{ if } X \leq S_T$$

or:

$$P_T = \max(0, X - S_T)$$

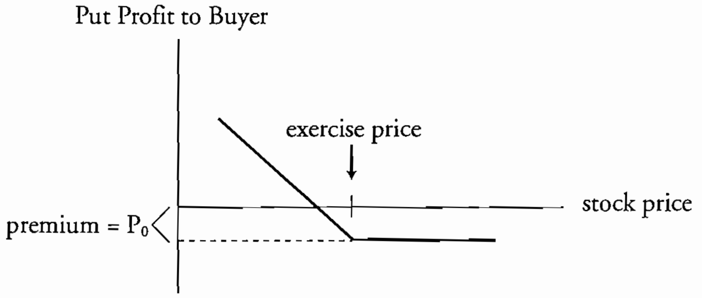
For example, an investor writes a put option on a stock with a strike price of X = 50. If the stock stays at $50 or above, the payoff of the put option is zero (because the holder may receive the same or better price by selling the underlying asset on the market rather than exercising the option). But if the stock price falls below $50, say to $40, the put option may be exercised with the option holder buying the stock from the market at $40 and selling it to the put writer at $50, for a $10 gain. The writer of the put option must pay the put price of $50, when it can be sold in the market at only $40, resulting in a $10 loss. The gain to the option holder is the same magnitude as the loss to the option writer. Figure 3 illustrates this example, excluding the initial cost of the put and transaction costs. Figure 4 includes the cost of the put (but not transaction costs) and illustrates the profit to the put owner.

Figure 3: Put Payoff to Buyer and Seller



Given the "mirror image quality" that results from the "zero-sum game" nature of options, we often just draw the profit to the buyer as shown in Figure 4. Then, we can simply remember that each positive (negative) value is a negative (positive) value for the seller.

Figure 4: Put Profit to Buyer



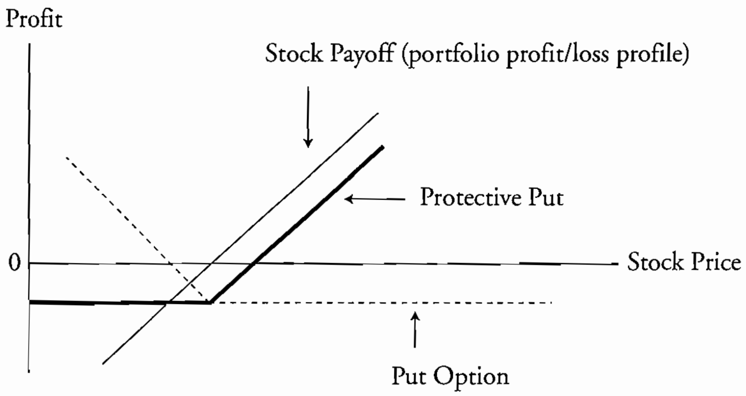
The breakeven price for a put position upon expiration is the exercise price minus the premium paid, $X - P_0$.

Covered Calls and Protective Puts

AIM 31.1: Explain the motivation to initiate a covered call or a protective put strategy.

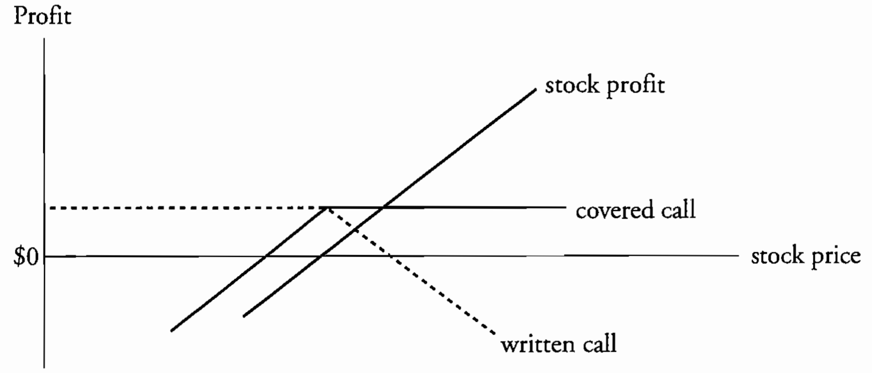
When an at-the-money long put position is combined with the underlying stock, we have created a **protective put** strategy. A protective put (also called *portfolio insurance* or a *hedged portfolio*) is constructed by holding a long position in the underlying security and buying a put option. You can use a protective put to limit the downside risk at the cost of the put premium, P_0. You will see by the diagram in Figure 5 that the investor will still be able to benefit from increases in the stock's price, but it will be lower by the amount paid for the put, P_0. Notice that the combined strategy looks very much like a call option. This should not be surprising since put-call parity requires that $p + S_0$ be the same as $c + Xe^{-rT}$. Figure 5 illustrates this property.

Figure 5: Protective Put Strategy



Another common strategy is to sell a call option on a stock that is owned by the option writer. This is called a **covered call** position. By writing an out-of-the-money call option, the combined position caps the upside potential at the strike price. In return for giving up any potential gain beyond the strike price, the writer receives the option premium. This strategy is used to generate cash on a stock that is not expected to increase above the exercise price over the life of the option.

Figure 6: Profit Profile for a Covered Call



Spread Strategies

AIM 31.2: Describe and explain the use and payoff functions of spread strategies, including bull spread, bear spread, box spread, calendar spread, butterfly spread, and diagonal spread.

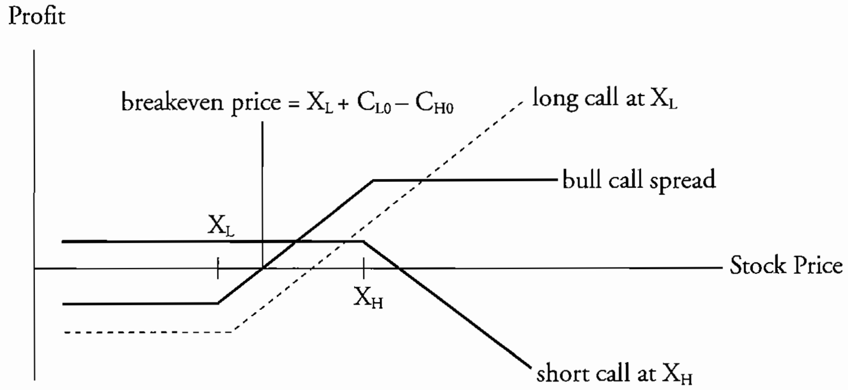
AIM 31.3: Calculate the pay-offs of various spread strategies.

Several spread strategies exist. These strategies combine options positions to create a desired payoff profile. The differences between the options are either the strike prices and/or the time to expiration. We will discuss bull and bear spreads, butterfly spreads, calendar spreads, and diagonal spreads.

Bull and Bear Spreads

In a *bull call spread,* the buyer of the spread purchases a call option with a low exercise price, X_L, and subsidizes the purchase price of the call by selling a call with a higher exercise price, X_H. The buyer of a bull call spread expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call.

Figure 7: Bull Call Spread



Example: Bull call spread

An investor purchases a call for C_{L0} = \$3.00 with a strike of X = \$40 and sells a call for C_{H0} = \$1.00 with a strike price of \$50. Compute the payoff of a bull call spread strategy when the price of the stock is at \$45.

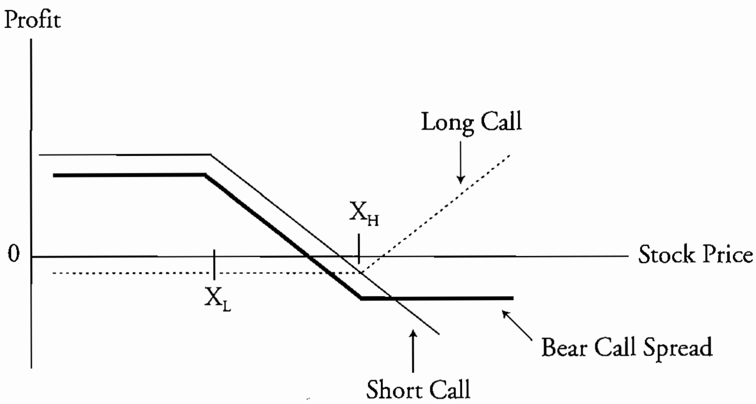
Answer:

$$\text{profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$$

$$\text{profit} = \max(0, 45 - 40) - \max(0, 45 - 50) - 3 + 1 = \$3.00$$

A *bear call spread* is the sale of a bull spread. That is, the bear spread trader will purchase the call with the higher exercise price and sell the call with the lower exercise price. This strategy is designed to profit from falling stock prices (i.e., a "bear" strategy). As stock prices fall, the investor keeps the premium from the written call, net of the long call's cost. The purpose of the long call is to protect from sharp increases in stock prices. The payoff is the opposite (mirror image) of the bull call spread and is shown in Figure 8.

Figure 8: Bear Call Spread



Puts can also be used to replicate the payoffs for both a bull call spread and a bear call spread. In a *bear put spread* the investor buys a put with a higher exercise price and sells a put with a lower exercise price.

Example: Bear put spread

An investor sells a put for P_{L0} = \$3.00 with a strike of X = \$20 and purchases a put for P_{H0} = \$4.50 with a strike price of \$40. Compute the payoff of a bear put spread strategy when the price of the stock is at \$35.

Answer:

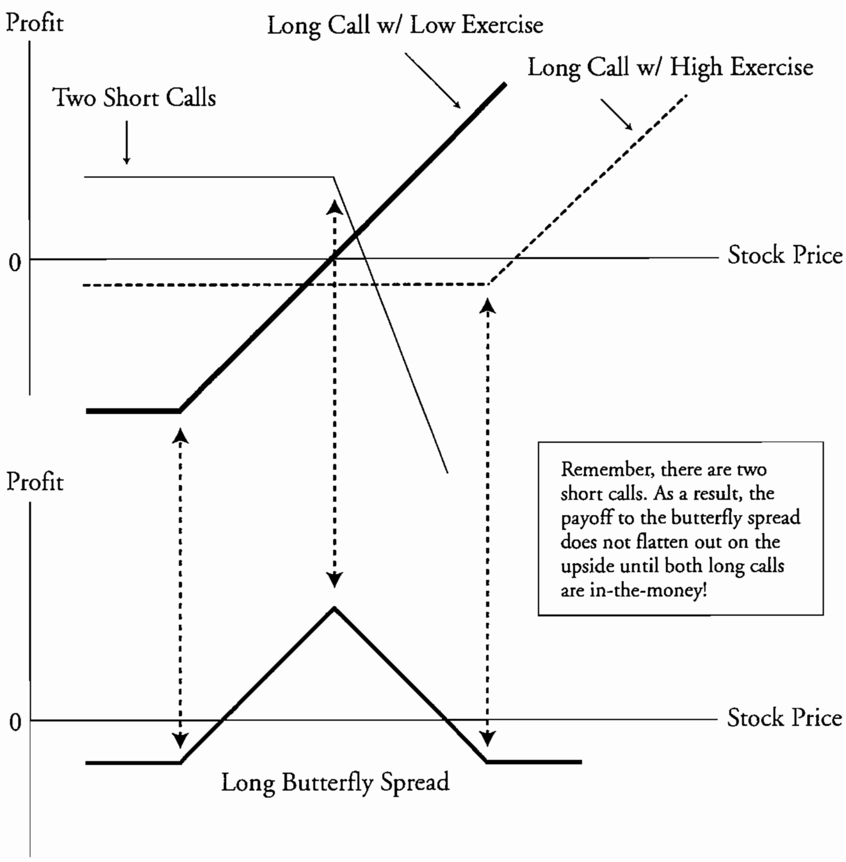
$$\text{profit} = \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$$

$$\text{profit} = \max(0, 40 - 35) - \max(0, 20 - 35) - 4.50 + 3 = \$3.50$$

Butterfly Spreads

A *butterfly spread* involves the purchase or sale of *three* different call options. Here, the investor buys one call with a low exercise price, buys another call with a high exercise price, and sells *two* calls with an exercise price in between. The buyer of a butterfly spread is essentially betting that the stock price will stay near the strike price of the written calls. However, the loss that the butterfly spread buyer sustains if the stock price strays from this level is limited. The two graphs in Figure 9 illustrate the construction and payoffs of a butterfly spread.

Figure 9: Butterfly Spread Construction and Behavior



Example: Butterfly spread with calls

An investor makes the following transactions in calls on a stock:

- Buys one call defined by C_{L0} = \$ 7.00 and X_L = \$55.
- Buys one call defined by C_{H0} = \$2.00 and X_H = \$65.
- Sell two calls defined by C_{M0} = \$4.00 and X_M = \$60.

Compute the payoff of a butterfly spread strategy with calls when the stock is at \$60.

Answer:

$$\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

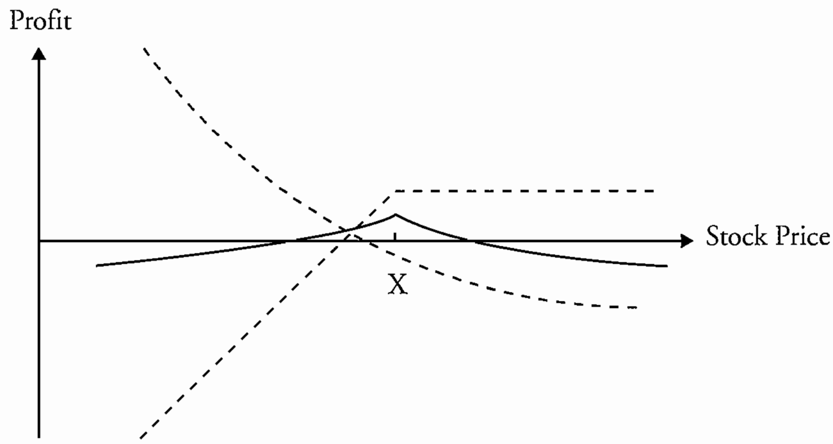
$$\text{profit} = \max(0, 60 - 55) - 2\max(0, 60 - 60) + \max(0, 60 - 65) - 7 + 2(4) - 2 = \$4.00$$

To create a butterfly spread with put options, the investor would buy a low and high strike put option and sell two puts with an intermediate strike price. Again, the combined position is constructed by summing the payoffs of the individual options at each stock price.

Calendar Spreads

A *calendar spread* is created by transacting in two options that have the same strike price but different expirations. Figure 10 shows a calendar spread using put options. The strategy sells the short-dated option and buys the long-dated option. Notice that the payoff here is similar to the butterfly spread. The investor profits only if the stock remains in a narrow range, but losses are limited. In this case, the losses are not symmetrical as they are in the butterfly spread. A calendar spread based on calls is created in similar fashion.

Figure 10: Calendar Spread



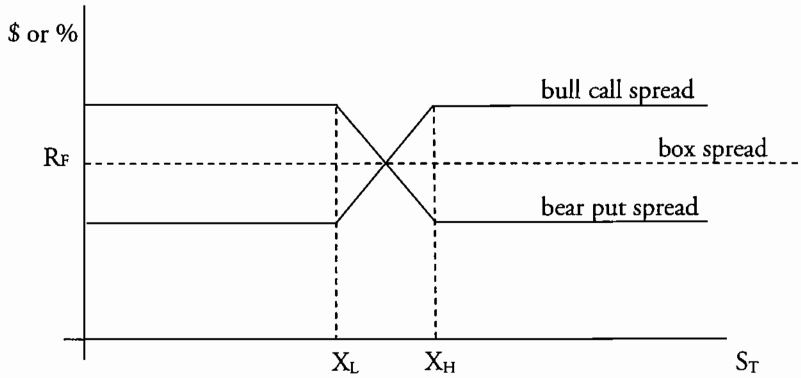
Diagonal Spreads

A *diagonal spread* is similar to a calendar spread except that instead of using options with the same strike price and different expirations, the options in a diagonal spread can have different strike prices in addition to different expirations.

Box Spreads

A **box spread** is a combination of a bull call spread and a bear put spread on the same asset. The payoff to the box spread is always the same, so if the options are priced correctly, the payoff must be the risk-free rate.

Figure 11: Payoff to the Box Spread



Combination Strategies

AIM 31.4: Describe and explain the use and payoff functions of combination strategies, including straddles, strangles, strips, or straps.

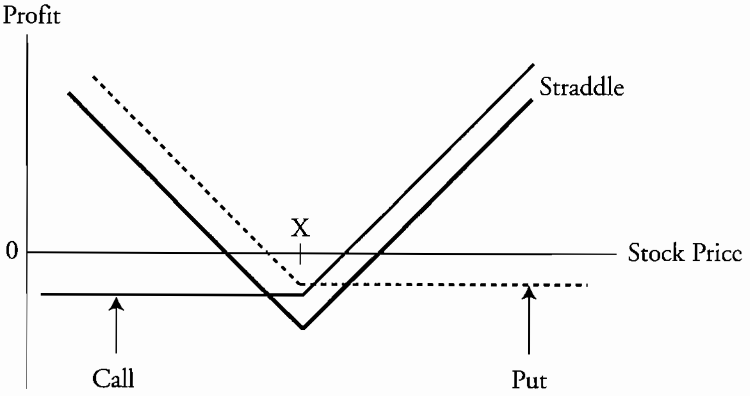
AIM 31.5: Compute the pay-offs of combination strategies.

Combinations are option strategies involving both puts and calls. We will discuss straddles, strangles, strips, and straps.

Straddle

A long *straddle* (bottom straddle or straddle purchase) is created by purchasing a call and a put with the same strike price and expiration. Figure 12 illustrates the payoff for a long straddle position. Both options have the same exercise price and expiration. Note that this strategy is profitable when the stock price moves strongly in either direction. This strategy bets on volatility. A short straddle (top straddle or straddle write) sells both options and bets on little movement in the stock. A short straddle bets on the same thing as the butterfly spread or the calendar spread, except the losses are not limited. It is a bet that will profit more if correct but also lose more if it is incorrect. Straddles are symmetric around the strike price.

Figure 12: Long Straddle Profit/Loss



Example: Straddle

An investor purchases a call on a stock, with an exercise price of \$45 and a premium of \$3, and purchases a put option with the same maturity that has an exercise price of \$45 and a premium of \$2. Compute the payoff of a straddle strategy if the stock is at \$35.

Answer:

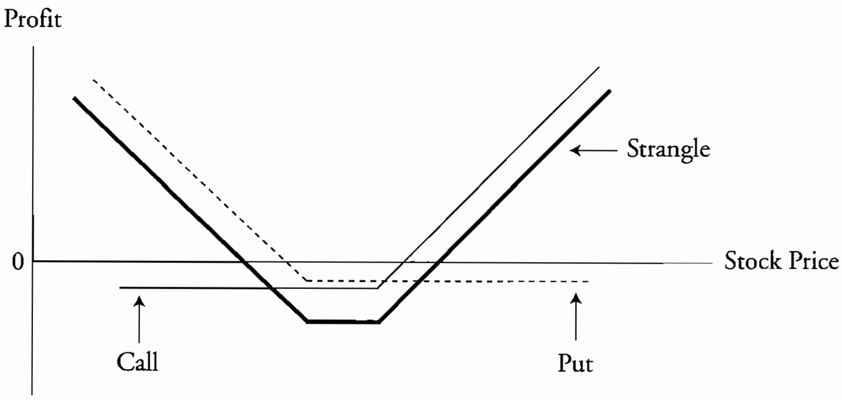
$$\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$$

$$\text{profit} = \max(0, 35 - 45) + \max(0, 45 - 35) - 3 - 2 = \$5$$

Strangle

A *strangle* (or bottom vertical combination) is similar to a straddle except that the options purchased are slightly out-of-the-money, so it is cheaper to implement than the straddle. The payoff is similar to the straddle except for a flat section between the strike prices, as shown in Figure 13. Because it is cheaper, the stock will have to move more relative to the straddle before the strangle pays off. Strangles are also symmetric around the strikes.

Figure 13: Long Strangle Profit/Loss



Example: Strangle

An investor purchases a call on a stock, with an exercise price of $50 and a premium of $1.50, and purchases a put option with the same maturity that has an exercise price of $45 and a premium of $2. Compute the payoff of a strangle strategy if the stock is at $40.

Answer:

$$\text{profit} = \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$$

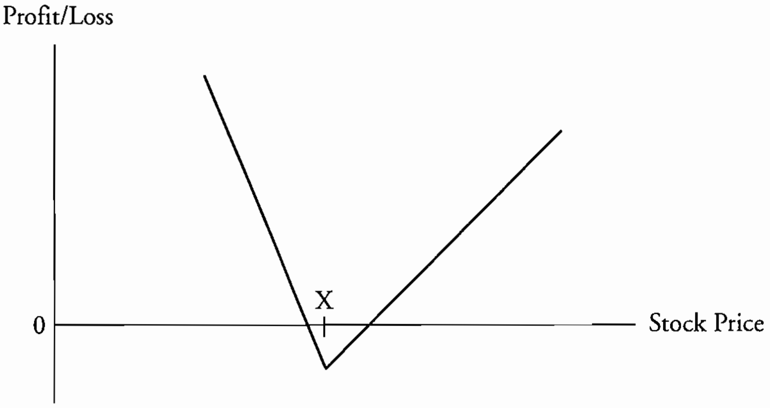
$$\text{profit} = \max(0, 40 - \$50) + \max(0, 45 - 40) - 1.50 - 2 = \$1.50$$

A short strangle (or a top vertical combination) is similar to the short straddle.

Strips and Straps

A *strip* involves purchasing two puts and one call with the same strike price and expiration. Figure 14 illustrates a strip. Notice the asymmetry of the payoff. A strip is betting on volatility but is more bearish since it pays off more on the downside.

Figure 14: Strip Profit/Loss

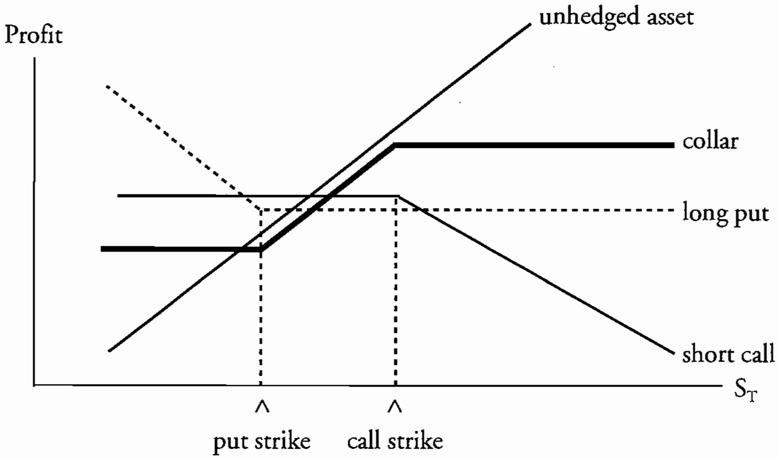


A *strap* involves purchasing two calls and one put with the same strike price and expiration. A strap is betting on volatility but is more bullish since it pays off more on the upside.

Collar

A **collar** is the combination of a protective put and covered call. The usual goal is for the owner of the underlying asset to buy a protective put and then sell a call to pay for the put. If the premiums of the two are equal, it is called a **zero-cost collar.**

Figure 15: Collar



Key Concepts

1. Stock options can be combined with their underlying stock to generate various payoff profiles. A protective put combines an at-the-money long put position with the underlying stock. A covered call involves selling a call option on a stock that is owned by the option writer.
2. Spread strategies combine options in the same option class to generate various payoff profiles. Spread strategies include bull, bear, butterfly, and calendar spreads.
3. Combination strategies combine puts and calls to generate various payoff strategies. The most common combination strategies include straddles, strangles, strips, and straps.
4. The buyer of a bull call spread expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call.
5. The bear call spread trader will purchase the call with the higher exercise price and sell the call with the lower exercise price. This strategy is designed to profit from falling stock prices (i.e., a "bear" strategy). As stock prices fall, the investor keeps the premium from the written call, net of the long call's cost.
6. The buyer of a butterfly spread is essentially betting that the stock price will stay near the strike price of the written calls. However, the loss that the butterfly spread buyer sustains if the stock price strays from this level is not large.
7. A box spread is an extreme method of locking in value. The dollar return for a box spread is fixed. It is a combination of a bull call spread and a bear put spread.
8. A long straddle (bottom straddle or straddle purchase) is created by purchasing a call and a put with the same strike price and expiration. Note that this strategy only pays off when the stock moves in either direction.
9. A strangle (or bottom vertical combination) is similar to a straddle except that the option purchased is slightly out-of-the-money, so it is cheaper to implement than the straddle.
10. A strip is betting on volatility but is more bearish since it pays off more on the down side.
11. A strap is betting on volatility but is more bullish since it pays off more on the up side.

Concept Checkers

1. An investor is very confident that a stock will change significantly over the next few months; however, the direction of the price change is unknown. Which strategies will most likely produce a profit if the stock price moves as expected?

   I. Short butterfly spread.
   II. Bearish calendar spread.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. Which of the following will create a bear spread?
   A. Buy a call with a strike price of X = 45 and sell a call with a strike price of X = 50.
   B. Buy a call with a strike price of X = 50 and buy a put with a strike price of X = 55.
   C. Buy a put with a strike price of X = 45 and sell a put with a strike price of X = 50.
   D. Buy a call with a strike price of X = 50 and sell a call with a strike price of X = 45.

3. An investor believes that a stock will either increase or decrease greatly in value over the next few months, but believes a down move is more likely. Which of the following strategies will be the best for this investor?
   A. A protective put.
   B. An at-the-money strip.
   C. An at-the-money strap.
   D. A top vertical combination.

4. An investor constructs a long straddle by buying an April \$30 call for \$4 and buying an April \$30 put for \$3. If the price of the underlying shares is \$27 at expiration, what is the profit on the position?
   A. –\$4.
   B. –\$2.
   C. \$2.
   D. \$3.

5. Consider an option strategy where an investor buys one call option with an exercise price of \$55 for \$7, sells two call options with an exercise price of \$60 for \$4, and buys one call option with an exercise price of \$65 for \$2. If the stock price declines to \$25, what will be the profit or loss on the strategy?
   A. –\$3.
   B. –\$1.
   C. \$1.
   D. \$2.

Concept Checker Answers

1. **A** A short butterfly spread will produce a modest profit if there is a large amount of volatility in the price of the stock. A bearish calendar spread is a play using options with different expiration dates.

2. **D** Spread strategies involve purchasing and selling an option of the same type. A bear spread with calls involves buying a call with a high strike price and selling a call with a low strike price. The investor profits if stock prices fall by keeping the premium from the written call, net of the premium from the purchased call. Note that a bear spread can also be constructed with put options by buying a put with a high strike price and selling a put with a low strike price. With a bear put spread, if the stock price declines and both puts are exercised, the investor receives the difference between the strike prices less the net premium paid.

3. **B** An at-the-money strip bets on volatility but is more bearish since it pays off more on the downside.

4. **A** The sum of the premiums paid for the position is \$7. With the underlying stock at \$27, the put will be worth \$3, while the call option will be worthless. The value of the position is (–\$7 + \$3) = –\$4.

5. **B** The strategy described is a butterfly spread where the investor buys a call with a low exercise price, buys another call with a high exercise price, and sell two calls with a price in between. In this case, if the option moves to \$25, none of the call options will be in the money, so the profit is equal to the net premium paid, which is –\$7 + (2 × \$4) – \$2 = –\$1.

Interest Rate Caps and Floors

Exam Focus

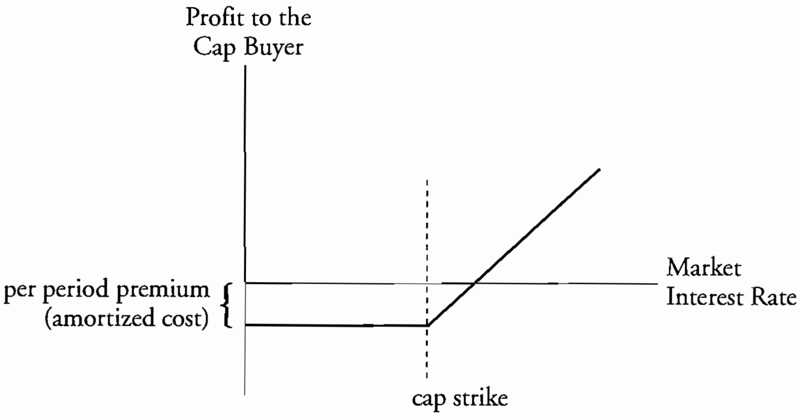
This optional reading addresses interest rate caps and floors, which were not covered in prior readings. Make sure you can calculate the payoff to a cap and a floor and explain how caps and floors are equivalent to packages of other securities.

An **interest rate cap** is an agreement in which one party agrees to pay the other at regular intervals over a certain period of time when the benchmark interest rate (e.g., LIBOR) exceeds the strike rate specified in the contract. This strike rate is called the **cap rate**. For example, the seller of a cap might agree to pay the buyer at the end of any quarter over the next two years if LIBOR is greater than a cap rate of 6%.

The buyer of a cap has a position similar to that of a buyer of a call on LIBOR, both of whom benefit when interest rates rise. Because an interest rate cap is a multi-period agreement, a cap is actually a portfolio of call options on LIBOR called **caplets**. For example, the 2-year cap discussed above is actually a portfolio of eight interest rate options with different maturity dates.

The cap buyer pays a premium to the seller and exercises the cap if the market rate of interest rises above the cap strike. The diagram in Figure 1 illustrates the profits of an interest rate cap at the end of one particular settlement period. It has the familiar shape of a long position in a call option.

Figure 1: Profit to a Long Cap

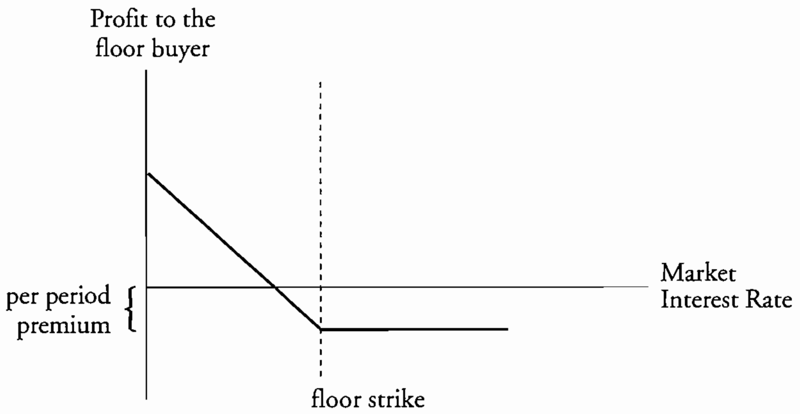


An **interest rate floor** is an agreement in which one party agrees to pay the other at regular intervals over a certain time period when the benchmark interest rate (e.g., LIBOR) falls below the strike rate specified in the contract. This strike rate is called the **floor rate**. For example, the seller of a floor might agree to pay the buyer at the end of any quarter over the next two years if LIBOR is less than a floor rate of 4%.

The buyer of a floor benefits from an interest rate decrease and, therefore, has a position that is similar to that of a buyer of a put on LIBOR, who benefits when interest rates fall and the price of the instrument rises. Once again, because a floor is a multi-period agreement, a floor is actually a portfolio of put options on LIBOR called **floorlets**.

The floor buyer pays a premium and exercises the floor if the market rate of interest falls below the floor strike. The diagram in Figure 2 illustrates the profits of an interest rate floor at the end of one particular settlement period. It has the same shape as a long put option.

Figure 2: Profit to a Long Floor



Options are traded both on *interest rates* and on *prices* of fixed-income securities. So far we've talked about options on interest rates. The values of comparable options on rates and prices respond differently to changes in interest rates because of the inverse relationship between bond yields and bond prices. Figure 3 outlines how each type of option responds to changes in yields and bond prices.

Figure 3: Options on Rate vs. Options on Prices

Option	*If Rates* Increase *and Bond Prices* Decrease	*If Rates* Decrease *and Bond Prices* Increase
Value of call on LIBOR	Increases	Decreases
Value of call on bond price	Decreases	Increases
Value of put on LIBOR	Decreases	Increases
Value of put on bond price	Increases	Decreases

We can also interpret caps and floors in terms of options on the prices of fixed-income securities:

- A long cap is equivalent to a portfolio of long put options on fixed-income security prices.
- A long floor is equivalent to a portfolio of long call options on fixed-income security prices.

A floating-rate borrower can use a cap to limit interest expense during the life of the cap. The payoff to the cap buyer is:

$$\text{periodic payment} = \max\left[0, (\text{notional principal}) \times (\text{index rate} - \text{cap strike}) \times \left(\frac{\text{actual days}}{360}\right)\right]$$

A floating-rate investor can use a floor to limit reductions in interest income during the life of the floor. The payment to the floor buyer is:

$$\text{periodic payment} = \max\left[0, (\text{notional principal}) \times (\text{floor strike} - \text{index rate}) \times \left(\frac{\text{actual days}}{360}\right)\right]$$

Example: Calculating the payoff for an interest rate cap

Suppose that a 1-year cap has a cap rate of 8% and a notional amount of $100 million. The frequency of settlement is quarterly, and the reference rate is 3-month LIBOR. Assume that 3-month LIBOR at the end of the next four quarters is as shown in Figure 4. Calculate the payoff for each quarter.

Figure 4: Payoff to 8% Interest Rate Cap

Quarter	*3-month LIBOR*	*Payoff*
1	7.7%	?
2	8.0%	?
3	8.4%	?
4	8.6%	?

Answer:

The cap will have a payoff each quarter equal to:

$$\max\left[0, \$100{,}000{,}000 \times \left(\frac{\text{LIBOR} - 0.08}{4}\right)\right]$$

Professor's Note: The division by 4 comes from the settlement frequency, assuming exactly 90 days per quarter.

$$\text{quarter 1 payoff} = \max\left[0,\ \$100{,}000{,}000 \times \left(\frac{0.077 - 0.08}{4}\right)\right] = \$0$$

$$\text{quarter 2 payoff} = \max\left[0,\ \$100{,}000{,}000 \times \left(\frac{0.08 - 0.08}{4}\right)\right] = \$0$$

$$\text{quarter 3 payoff} = \max\left[0,\ \$100{,}000{,}000 \times \left(\frac{0.084 - 0.08}{4}\right)\right] = \$100{,}000$$

$$\text{quarter 4 payoff} = \max\left[0,\ \$100{,}000{,}000 \times \left(\frac{0.086 - 0.08}{4}\right)\right] = \$150{,}000$$

Example: Calculating the payoff for an interest rate floor

Let's change the cap to a *floor*, keeping the other information the same. **Calculate** the payoff for a floor with a floor rate of 8%.

Answer:

$$\text{quarter 1 payoff} = \max\left[0,\ \$100{,}000{,}000 \times \left(\frac{0.08 - 0.077}{4}\right)\right] = \$75{,}000$$

$$\text{quarter 2 payoff} = \max\left[0,\ \$100{,}000{,}000 \times \left(\frac{0.08 - 0.08}{4}\right)\right] = \$0$$

$$\text{quarter 3 payoff} = \max\left[0,\ \$100{,}000{,}000 \times \left(\frac{0.08 - 0.084}{4}\right)\right] = \$0$$

$$\text{quarter 4 payoff} = \max\left[0,\ \$100{,}000{,}000 \times \left(\frac{0.08 - 0.086}{4}\right)\right] = \$0$$

As a technical note, caps and floors are usually paid in arrears, meaning that, for example, the $75,000 payoff to the floor for the first quarter will be made three months later, at the end of the second quarter.

An **interest rate collar** is a simultaneous position in a floor and a cap on the same benchmark rate over the same period with the same settlement dates. There are two types of collars:

- The first type of collar is to purchase a cap and sell a floor. For example, an investor with a LIBOR-based liability could purchase a cap on LIBOR at 8% and simultaneously sell a floor on LIBOR at 4% over the next year. The investor has now hedged the liability so that the borrowing costs will stay within the "collar" of 4% to 8%. If the cap and floor rates are set so that the premium paid from buying the cap is exactly offset by the premium received from selling the floor, the collar is called a "zero-cost" collar.
- The second type of collar is to purchase a floor and sell a cap. For example, an investor with a LIBOR-based asset could purchase a floor on LIBOR at 3% and simultaneously sell a cap at 7% over the next year. The investor has now hedged the asset so the returns will stay within the collar of 3% to 7%. The investor can create a zero-cost collar by choosing the cap and floor rates so that the premium paid on the floor offsets the premium received on the cap.

The following is a review of the Financial Markets and Products principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Fundamentals of Commodity Spot and Futures Markets: Instruments, Exchanges and Strategies

Topic 32

Exam Focus

This topic is an introduction to commodities markets and will help you understand commodity pricing, which is outlined in the next topic. Here we will discuss the major players in the commodity futures market and the risks involved with trading in this market. For the exam, understand the concept of basis risk and know how to calculate the effectiveness of a futures hedge. Also, be familiar with the major risks involved with trading in the commodity spot and futures markets.

The use of futures and forwards to hedge commodity and financial product price risk has become a key component of modern risk management. It is interesting to note that forwards and futures are deeply rooted in U.S. history. In the early years of the United States, rivers were the primary means of transportation. With this in mind, the major trade cities grew up along waterways. Among these cities prominent in commodities and trade were Chicago, New Orleans, Kansas City, and New York.

The major industry in the United States at that time was agriculture. Without any "forward" means of establishing prices, the price structure was very cyclical in nature. Production was high during harvest time and prices, due to the high supply of harvest, were correspondingly low. As the months progressed and time grew further away from harvest, supplies started to diminish and prices started to rise. As a result, prices were low at harvest time and high in the months thereafter. This method of commodity pricing was not very functional, as neither the farmer, nor the processor, could reasonably establish their prices, as they were constantly on opposite sides of the pricing structure. In addition to the commodity prices being dependent on the time of the year, the prices were also dependent on outside influences, such as weather and politics. Clearly, something had to be done to somehow stabilize these highly seasonal, cyclical pricing features of commodities.

In the early part of the 19th century, in an attempt to alleviate pricing risks, "cash forward" contracts were conceived in order to provide some price stability to the commodity marketplace. In a privately negotiated transaction, the farmer and the user would reach an agreement for the user to purchase all, or part, of the farmer's crop at a pre-arranged price when harvested. These cash forward contracts provided a means in which the producer (farmer) was able to sell his product (crop) for a specified price for delivery at a later time. The time periods involved were usually fairly short. This "pre-determination" of price allowed the buyer and seller to negotiate based on the fact that they would finally have some knowledge of the price they would pay or receive, as well as the timetable involved.

In 1922, Congress enacted the Grain Futures Act, which was initially set up to regulate futures exchanges. This regulatory action was followed up by the Commodity Futures Trading Act of 1974 which established the Commodity Futures Trading Commission (CFTC) as the responsible entity for the regulation of futures exchanges.

Among its many duties, the CFTC is the body that approves futures trading at the exchange level and at the contract level. Any new exchange that wishes to be created must have approval from the CFTC. The introduction of new contracts or changes to existing contracts must also go through CFTC approval before being effectuated. The primary role of the CFTC is to assure that fair and orderly futures transactions occur in the public interest. Also, futures trading must have a justifiably useful economic purpose as opposed to being a form of legal gambling. In sum, the purpose of the CFTC is to maintain the effectiveness and integrity of futures trading.

Differences Between Spot, Forward, and Futures

AIM 32.4: Explain the major differences between spot, forward, and futures transactions, markets, and contracts.

The most basic type of futures or forward contract obligates one counterparty to buy and the other counterparty to sell a given asset at a given price. Comparing forward and futures contracts to spot contracts is a useful first step in understanding these derivatives.

Spot contracts are used when a seller of an asset agrees to deliver the asset immediately, and the buyer agrees to purchase the asset immediately. Therefore, a **spot price** is the price of an asset for immediate delivery.

Forward contracts are contracts in which the buyer and seller agree on a price and quantity today, but delivery does not occur until some pre-specified date in the future. A forward contract is non-standardized (i.e., trades OTC) in that the contract terms and provisions are defined only by the parties to the contract, with no third-party intervention. The person selling forward is obligated to make delivery; the person buying forward is obligated to take delivery. There is a risk that either party may ultimately break the commitment.

Futures contracts are essentially forward contracts that are arranged by an organized exchange. Futures contracts usually require a margin deposit. This position is marked to market daily.

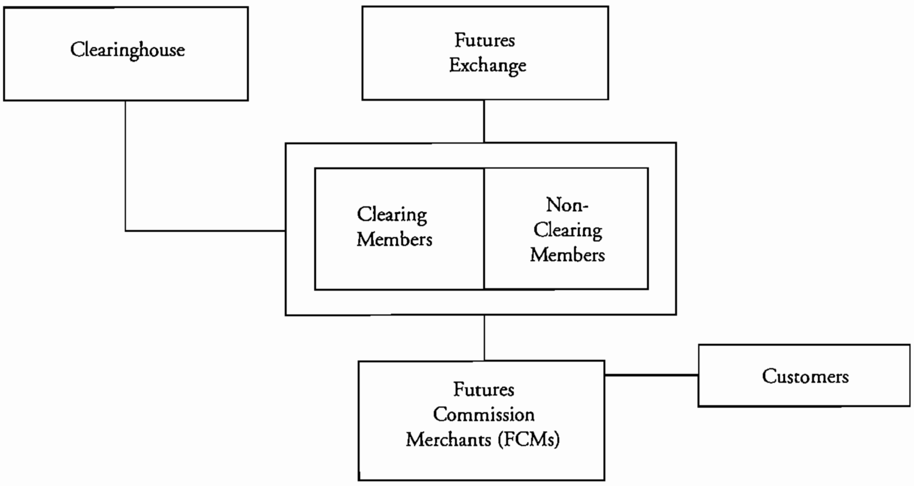
Futures contracts are standardized according to the guidelines of the exchange. Forwards are bilateral contracts subject to counterparty default risk, while counterparty default risk on futures contracts is essentially eliminated because the futures exchange clearinghouse guarantees the transaction.

AIM 32.7: Describe the structure of the futures market.

Clearinghouses enable futures positions to be offset easily prior to delivery. To offset, close, or liquidate a futures position before delivery, an investor must complete a transaction opposite to the trade that initiated (i.e., opened) the futures position. The offsetting transaction must occur in the same commodity, for the same delivery month, and on the same exchange. About 98% of futures contracts are offset before delivery.

The connection among key players in futures markets is shown in Figure 1:

Figure 1: Futures Market Structure



The key role of a futures exchange is to provide a location for **price discovery**. Only **exchange members** may execute trades on commodity exchange trading floors. **Floor brokers** have relationships with commodity brokerage firms [also known as futures commission merchants (FCMs)] and fill orders for the FCM's customers. **Floor traders** (also known as locals) trade on the floor for their own accounts.

Bill of Lading

AIM 32.1: Define "bill of lading."

Regarding the physical exchange of goods, the **bill of lading** (BOL) is a document that specifies the commodity owner and acknowledges that goods have been received as cargo and are ready for transportation. This document also includes items such as ports of departure, package weights, and dates of shipment.

Commodity Spot Markets

AIM 32.2: Define the major risks involved with commodity spot transactions.

AIM 32.3: Differentiate between ordinary and extraordinary transportation risks.

The major risks involved with commodity spot transactions are as follows:

Price risk: Owner of actuals (i.e., the underlying commodity) bears the risk of the commodity falling in value. This risk may be largely reduced by trading in forwards or futures contracts.

Transportation risk: This risk is divided into two areas: ordinary (i.e., deterioration, spoilage, accident) and extraordinary (i.e., acts of God, wars, riots, strikes). For example, transportation risk could result from ships lost at sea or train cars overturned.

In either the case of ordinary or extraordinary, the **free-on-board** (FOB) buyer is responsible for transportation risk. Given this risk, it would be wise to purchase insurance. If a trade settles for future delivery with the cost of insurance and freight (CIF) prices factored in, a seller could hedge the position against a decline in price by selling futures but also enter into a Forward Freight Agreement to hedge the associated transportation costs.

Delivery risk: Parties may renege on delivery. This risk in modern times has been greatly diminished due to clearinghouses and robust contracts.

Credit risk: An unreliable counterparty makes this an issue primarily for the spot market.

Margin requirements are in place for all exchange-traded futures contracts. These requirements are mainly put in place to address liquidity concerns. If liquidity decreases, the margin requirements placed by lenders may increase. If a trader cannot fund his position, he may be forced at an inopportune time to unwind the position.

Futures Market Participants

AIM 32.5: Describe the basic characteristics and differences between hedgers, speculators, and arbitrageurs.

Hedgers

Futures markets were established essentially by farmers and ranchers who wanted to lock-in a fixed price in advance for their crops or livestock. In some ways not much has changed, as they still do this today. In addition, users of commodities such as airlines also hedge in a similar fashion.

For example, suppose that Sky High Airlines knows in June that it will have to buy 10 million tons of jet fuel in November. The airline will buy futures to hedge the possibility of a price increase between June and November. With futures, the hedger effectively locks-in the price to be paid in advance.

A hedged position is noticeably less risky than an unhedged position. In an **unhedged position**, an investor can lose the full value of the commodity if its price decreases to zero. If the position is hedged, the investor has acquired a degree of protection against any adverse movements.

Speculators

Speculators accept the price risk that hedgers are unwilling to bear. For a commodity producer/acquirer to hedge its commodity price risk by buying or selling futures, there must be someone on the other end of the trade willing to buy futures and accept the price risk.

For example, an investor with risk capital may decide to take a position in a futures contract on oil and by doing so creates exposure to oil prices. He is betting that oil prices will move in his favor. If they work against him, the investor could lose all that he invested plus more. The other side of the contract, the hedge for example, would be the winner in this case. The futures hedger transfers commodity price risk to the futures speculator. The speculator accepts the risk of changing prices, whereas the hedger assumes only the risk of a change in basis (as we will discuss in AIM 32.8).

Arbitrageurs

Arbitrageurs are interested in market inefficiencies which allow them to obtain a riskless profit. For example, when the cash and futures price difference exceeds carrying charges, arbitrage opportunities exist. By buying the nearby contract and simultaneously selling a distant contract at a price that exceeds the carrying charges, the arbitrageur profits as prices return to their normal relationship. Arbitrage is the opportunity to profit from such temporary abnormal price differences and, when used correctly, is riskless.

Arbitrage Portfolio

AIM 32.6: Describe an "arbitrage portfolio" and explain the conditions for a market to be arbitrage free.

Arbitrage is an important concept in valuing (pricing) derivative securities. In its purest sense, arbitrage is riskless. If a return greater than the risk-free rate can be earned by holding a portfolio of assets that produces a certain (riskless) return, then an arbitrage opportunity exists.

Arbitrage opportunities arise when assets are mispriced. Trading by arbitrageurs will continue until they affect supply and demand enough to bring asset prices to efficient (no-arbitrage) levels. Once efficient levels are reached, the market is said to be arbitrage free.

There are two arbitrage arguments that are particularly useful in the study and use of derivatives.

The first is based on the **law of one price**. Two securities or portfolios that have identical cash flows in the future, regardless of future events, should have the same price. If A and B have the identical future payoffs, and A is priced lower than B, buy A and sell B. You will have an immediate profit, and the payoff on A will satisfy the (future) liability of being short on B.

The second type of arbitrage is used when two securities with uncertain returns can be combined in a portfolio that will have a certain payoff. If a portfolio consisting of A and B has a certain payoff, the portfolio should yield the risk-free rate. If this no-arbitrage condition is violated in that the certain return of A and B together is higher than the risk-free rate, an arbitrage opportunity exists. An arbitrageur could borrow at the risk-free rate, buy the A + B portfolio, and earn arbitrage profits when the certain payoff occurs. The payoff will be more than is required to pay back the loan at the risk-free rate.

Basis Risk

AIM 32.8: Define basis risk and the variance of the basis.

Basis is the difference between the cash or spot price of a commodity and the price of a futures contract on the same commodity at any given time, *t*. Although basis compares two prices that usually move in the same direction, these two prices do not often move by the same amount. The spot price can change over time, but F_0 is constant. Thus, at some time, *t*, the basis is $S_t - F_0$.

Changes in the basis are caused by changes in the cost of carry of the asset. Savvy investors analyze their risk not only at time *T*, but also at time *t*. **Basis risk** is the volatility of the basis over time and as a result is usually represented as the variance of the basis:

$$\sigma^2_{S(t)-F(t)} = \sigma^2_{S(t)} + \sigma^2_{F(t)} - 2\sigma_{S(t)}\sigma_{F(t)}\rho_{S,F}$$

This variance equation demonstrates that basis risk will be zero in the event that: (1) spot and futures variance are the same and (2) correlation between spot and futures is one.

AIM 32.9: Identify a commonly used measure for the effectiveness of hedging a spot position with a futures contract; use this measure to compute and compare the effectiveness of alternative hedges.

Hedgers try to reduce price risk to as close to zero as possible. The measure of the effectiveness of hedging the actuals with futures is defined by:

$$\text{hedge effectiveness} = 1 - \frac{\sigma^2_{S(t)-F(t)}}{\sigma^2_{S(t)}}$$

When the variance of the basis divided by the variance of the spot is close to zero, the hedge effectiveness measure will be close to one. High hedge effectiveness measures are indicative of effective hedges.

Exchange for Physicals vs. Alternative Delivery Procedure

AIM 32.10: Define and differentiate between an exchange for physicals and an alternative delivery procedure.

An **exchange for physicals** (EFP) can end a contract. Here, a counterparty finds a willing trader with an opposite position and delivers the goods in a transaction that takes place off the floor of the exchange. The two parties can negotiate the terms of the transaction. An EFP is an ex-pit transaction and is the one exception to the Federal law that requires that all trades take place on the floor of the exchange. The parties must then inform the clearinghouse as to what transpired.

An **alternative delivery procedure** deals with the buyer and seller coming up with new terms for the contract after they have been matched by the exchange. The parties in the transaction must submit a notice of intent to their respective exchange detailing the changing terms.

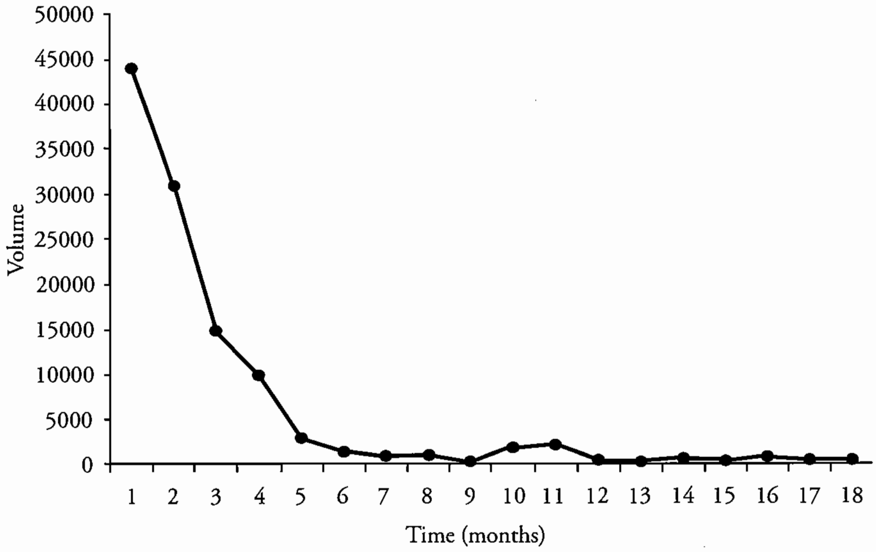
Market Depth

AIM 32.11: Describe volume and open interest and their relationship to liquidity and market depth.

Liquidity can be defined as low when it is difficult to raise money by selling an asset; when selling depresses the sale price. Keep in mind that speculators smooth price fluctuations and provide market liquidity. Speculators also finance their trades by borrowing from banks that set margin requirements. Since banks can reset margin, speculators face **funding liquidity risk** due to the risk of higher margins and/or losses on their existing positions.

Market depth may be discovered by measuring how many units traders can sell or buy at the current bid or ask price without moving the price. Typically, the highest levels of liquidity are found in the short-term. For example, Figure 2 shows an example of the average daily trading volume for crude oil futures contracts.

Figure 2: Futures Liquidity Over Time



The number of contracts that are exchanged each day is represented in the **volume** of each futures product. Typically, oil futures contracts [crude oil West Texas Intermediate (WTI)] are the most heavily traded contracts. Other heavily traded futures contracts include corn, soybeans, coffee, gas, aluminum, cooper, gold, and silver.

At any moment in time, the number of futures contracts outstanding is referred to as the **open interest**. The exchange publishes all open interest figures for the previous trading day. By examining open interest deviations, technical analysts are able to spot trends regarding the motivation of buyers and sellers to hold on to or to offset a particular commodity futures contract.

Key Concepts

1. Spot contracts are used when a seller of an asset agrees to deliver the asset immediately and the buyer agrees to purchase the asset immediately.

2. Forward contracts are contracts in which the buyer and seller agree on a price and quantity today, but delivery does not occur until some pre-specified date in the future.

3. Futures contracts are essentially forward contracts that are arranged by an organized exchange.

4. The bill of lading is a document that specifies the commodity owner and acknowledges that goods have been received.

5. The major risks involved with commodity spot transactions are price risk, transportation risk, delivery risk, and credit risk.

6. Basis risk refers to the risk that remains after a futures hedge has been implemented.

7. Hedgers use futures or forwards to lock-in a set price in advance. Speculators are on the other side of the transaction and accept the price risk that hedgers are unwilling to bear. Arbitrageurs are interested in market inefficiencies which allow them to obtain a riskless profit.

8. Riskless arbitrage involves earning more than the risk-free rate with no risk or earning an immediate gain with no future liability.

9. Basis is the difference between the spot price of a commodity and the price of a futures contract on the same commodity. Basis risk is the volatility of the basis over time.

10. Hedge effectiveness can be measured with the following equation:

$$\text{hedge effectiveness} = 1 - \frac{\sigma^2_{S(t)-F(t)}}{\sigma^2_{S(t)}}$$

    The closer this measure is to one, the more effective the hedge.

11. In an exchange for physicals, a counterparty finds a willing trader with an opposite position and delivers the goods in a transaction that takes place off the floor of the exchange. An alternative delivery procedure deals with the buyer and seller coming up with new terms for the contract.

12. Market depth can be found by measuring how many units traders can sell or buy at the current bid or ask price without moving the price.

Concept Checkers

1. Which of the following statements regarding spot commodity contracts is(are) true?
    I. Spot contracts are not traded on commodity futures exchanges.
    II. Storage and insurance costs are important considerations in owning the actuals.

    A. I only.
    B. II only.
    C. Both I and II.
    D. Neither I nor II.

2. Each of the following are major risks involved with commodity spot transactions except:
    A. price risk.
    B. transportation risk.
    C. delivery risk.
    D. ex-pit risk.

3. Assume the holder of a long futures position negotiates off the floor of the exchange with the holder of a short futures position to accept delivery to close out both the long and short positions. Which of the following statements about the transaction is true? The transaction is:
    A. known as delivery.
    B. known as cash settlement.
    C. known as an exchange for physicals.
    D. not a legal transaction. U.S. Federal law requires that all transactions take place on the floor of the exchange.

4. A farmer hedges the price risk of his crop with a grain futures contract. The variance of the commodity futures price is 6% while the variance of the spot price on the same commodity is 7%. The variance of the difference between the spot price and the futures price is 1%. The hedge effectiveness measure from using this commodity futures contract is closest to:
    A. 0.86.
    B. 0.83.
    C. 0.14.
    D. 0.99.

5. Arbitrage prevents:
    A. risk management.
    B. market efficiency.
    C. profit higher than the risk-free rate of return.
    D. two assets with identical payoffs from selling at different prices.

Concept Checker Answers

1. **C** Futures contract trading only takes place on futures exchanges. The cost of insurance and storage are always important considerations.

2. **D** Price risk, transportation risk, and delivery risk are forms of risk faced by spot contract holders.

3. **C** When the holder of a long position negotiates directly with the holder of the short position to accept delivery of the underlying commodity to close out both positions, the transaction is called an exchange for physicals. An EFP is a private transaction that occurs ex-pit and is the one exception to the Federal law that all trades take place on the exchange floor. Note that the exchange for physicals differs from an offsetting trade in which no delivery takes place, and it also differs from delivery in which the commodity is simply delivered as a result of the futures expiration with no secondary agreement. Most futures positions are settled by an offsetting trade.

4. **A** Hedge effectiveness is calculated as:

   1 – [(variance of basis) / (variance of spot)] = 1 – (0.01/0.07) = 0.857

   Since this measure is close to one, it appears to be effective. However, this measure is more useful as a relative measure by comparing the effectiveness of different potential hedges.

5. **D** Arbitrage forces two assets with the same expected future value to sell for the same current price. If this were not the case, you could simultaneously buy the cheaper asset and sell the more expensive one for a guaranteed riskless profit.

The following is a review of the Financial Markets and Products principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Commodity Forwards and Futures

Topic 33

Exam Focus

This topic on commodity forwards and futures focuses on the pricing relationships that exist when commodities have characteristics such as lease rates, storage costs, and/or convenience yields. Before you begin this topic, recall the no-arbitrage pricing relationships for futures contracts that were discussed in Topic 27 (Determination of Forward and Futures Prices). You should understand the basic futures pricing equation and how it is adjusted for lease rates, storage costs, and/or convenience yields.

Pricing Commodity Forwards and Futures

AIM 33.1: Define forward strip and forward curve.

AIM 33.2: Describe how to create a synthetic commodity position and use it to explain the relationship between the forward price and the expected future spot price.

AIM 33.4: Derive the basic equilibrium formula for pricing commodity forwards and futures.

AIM 33.5: Explain the implication basic equilibrium has for different types of commodities.

Professor's Note: AIM 33.5 is addressed throughout this topic.

Commodity and financial forward contracts are similar in some regards. For example, the prices of both are logically based upon expected spot prices. Some financial forwards (e.g., S&P 500 Index) are based upon the expected future spot price minus dividends received during the holding period. The price of a commodity forward must also be based upon expectations, but there are several factors to consider. For example, based upon their physical qualities, some commodities are *storable* (e.g., metals) and the associated costs depend upon the physical characteristics of the commodity. Also, due to their physical nature, others are not storable (e.g., electricity, perishable foods).

Some commodities are also appropriate for *leasing*. That is, an investor without a current need purchases the commodity and then lends it out to others who do have a current need. Just as with the loan of any asset the lender requires a return, so a **lease rate** (i.e., required return) is established. For example, assume an investor uses cash and purchases a commodity. If a viable lease market exists for the commodity, the investor might lend it to someone. Since the investor used cash to acquire the commodity, he must charge a lease

rate. Failing to do so would amount to an interest-free loan of the money tied up in the commodity.

Since commodity forward prices are based upon expected spot prices and expected spot prices are, in turn, dependent upon expected supply and demand forces, forward prices for commodities need not be constant from period to period. There are factors such as weather that can affect expected supply. For example, severe weather might be expected to reduce future coffee supplies, so the forward coffee price might incorporate the expected shortage into an increased forward price. Demand for a commodity can also be subject to change. For example, demand for electricity is not constant during the day nor is it constant across different seasons of the year or in different locations across the country. Estimating the expected spot price for a commodity, therefore, must utilize forecasts of all relevant factors.

For a given commodity on any trading day, several futures contracts will exist with varying maturity dates. The prices of the commodity futures contracts will differ with the different contract expiration dates. The set of futures prices for a given commodity is known as a **forward curve** or a **forward strip** on that particular day.

Assume that we do not know the forward price of the commodity and wish to derive it. A synthetic commodity forward price can be derived by combining a long position on a commodity forward, $F_{0,T}$, and a long zero-coupon bond that pays $F_{0,T}$ at time T.

The total cost at time 0 is equivalent to the cost of the bond, $e^{-rT}F_{0,T}$, where r represents the risk-free rate of return. The forward contract does not have any initial cash flows at time 0. The payoff at time T will be the payoff from the forward contract $(S_T - F_{0,T})$ plus the payoff from the bond $(F_{0,T})$:

$$S_T - F_{0,T} + F_{0,T} = S_T$$

where:
S_T = spot price of the commodity at time T

The present value of the expected spot price at time T is $E(S_T)e^{-\alpha T}$, where α represents the discount rate for the S_T cash flow at time T. This amount is equivalent to the cost of the bond, $e^{-rT}F_{0,T}$, because both represent the amount you would pay today to receive the commodity at time T. This equality is expressed in the following equation:

$$e^{-rT}F_{0,T} = E(S_T)e^{-\alpha T}$$

This equation illustrates that when using a risk-free discount rate, the discounted commodity forward price at time T is equivalent to the present value of a unit of commodity received at time T.

Multiplying each side of the equation by e^{rT} allows us to express the commodity forward price as follows:

$$F_{0,T} = E(S_T)e^{(r-\alpha)T}$$

Thus, the forward price today is a biased estimate of the expected commodity spot price at time T. The bias is a function of the risk premium on the commodity, $r - \alpha$. This equation is used to calculate the net present value (NPV) of commodities with available forward prices.

Electricity Prices

AIM 33.3: Explain the effect non-storability has on electricity prices.

As previously mentioned, electricity is not a storable commodity. Once it is produced, it must be used or it will likely go to waste. In addition, demand for electricity is not constant and will vary with time of day, day of the week, and season. Given the non-storability characteristic of electricity, its price is set by demand and supply at a given point in time. Since arbitrage opportunities do not exist with electricity (i.e., the inability to buy electricity during one season and sell it during another season) futures prices on electricity will vary much more during the trading day than financial futures.

Commodity Arbitrage

AIM 33.6: Describe an arbitrage transaction in commodity forwards and futures, and compute the potential arbitrage profit.

A **cash-and-carry arbitrage** consists of buying the commodity, storing/holding the commodity, and selling the commodity at the futures price when the contract expires. The steps in a cash-and-carry arbitrage are as follows.

At the initiation of the contract:

- Borrow money for the term of the contract at market interest rates.
- Buy the underlying commodity at the spot price.
- Sell a futures contract at the current futures price.

At contract expiration:

- Deliver the commodity and receive the futures contract price.
- Repay the loan plus interest.

If the futures contract is overpriced, this 5-step transaction will generate a riskless profit. The futures contract is overpriced if the actual market price is greater than the no-arbitrage price.

Example: Futures cash-and-carry arbitrage

Assume the spot price of gold is $900/oz., that the 1-year futures price is $975/oz., and that an investor can borrow or lend funds at 7%. Ignore transaction and storage costs. **Calculate** the arbitrage profit.

Answer:

The futures price, according to the no-arbitrage principle, should be:

$$F_{0,T} = \$900e^{0.07} = \$965$$

Instead, it's trading at $975. That means the futures contract is overpriced, so we should conduct cash and carry arbitrage by going short in the futures contract, buying gold in the spot market, and borrowing money to pay for the purchase. If we borrow $900 to fund the purchase of gold, we must repay $965.

Today		*1 year from today*	
Spot price of gold	$900		
Futures price of gold	$975		
Transaction	*Cash flow*	*Transaction*	*Cash flow*
Short futures	$0	Settle short position by delivering gold	+$975
Buy gold in spot market	–$900		
Borrow at 7%	+$900	Repay loan	–$965
Total cash flow	$0	Total cash flow = arbitrage profit	+$10

The riskless profit is equal to the difference between the futures contract proceeds and the loan payoff, or $975 – $965 = $10. Notice that this profit is equal to the difference between the actual futures price of $975 and the no-arbitrage price of $965.

If the futures price is too low (which presents a profitable arbitrage opportunity), the opposite of each step should be executed to earn a riskless profit.

This is **reverse cash-and-carry arbitrage**. The steps in reverse cash-and-carry arbitrage are as follows.

At the initiation of the contract:

- Sell commodity short.
- Lend short sale proceeds at market interest rates.
- Buy futures contract at market price.

At contract expiration:

- Collect loan proceeds.
- Take delivery of the commodity for the futures price and cover the short sale commitment.

Example: Futures reverse cash-and-carry arbitrage

Assume gold is priced at $900/oz., that the 1-year futures price is $950/oz., and that an investor can borrow or lend funds at 7%. Ignore transaction and storage costs. Calculate the profits from arbitrage.

Answer:

The futures price, according to the no-arbitrage principle, should be:

$$F_{0,T} = \$900e^{0.07} = \$965$$

Instead, it's trading at $950. That means the futures contract is underpriced, so we should conduct reverse cash and carry arbitrage by going long in the futures contract, shorting gold, and investing the short-sale proceeds:

Today		*1 year from today*	
Spot price of gold	$900		
Futures price of gold	$950		
Transaction	*Cash flow*	*Transaction*	*Cash flow*
Long futures	$0	Settle long position by buying gold	–$950
Short gold	+$900	Deliver gold to close short position	
Invest short-sale proceeds at 7%	–$900	Receive investments proceeds	+$965
Total cash flow	$0	Total cash flow = arbitrage profit	+$15

The riskless profit is equal to the loan proceeds less the futures contract payment, or $965 – $950 = $15.

Professor's Note: It may help to remember "buy low, sell high." If the futures price is "too high," sell the future and buy the spot. If the futures price is "too low," buy the future and sell the spot.

Lease Rates

AIM 33.7: Define the lease rate and how it determines the no-arbitrage values for commodity forwards and futures, and explain the relationship between lease rates and contango and lease rates and backwardation.

A **lease rate** is the amount of interest a lender of a commodity requires. The lease rate is defined as the amount of return the investor requires to buy and then lend a commodity. From the borrower's perspective, the lease rate represents the cost of borrowing the commodity. The lease rate and risk-free rate are important inputs to determine the commodity forward price. The lease rate in the pricing of a commodity forward is very similar to the dividend payment in a financial forward.

A no-arbitrage price can be established if there is an active lending market for a commodity. A commodity lender can earn a return, the lease rate, by buying a commodity and immediately selling it forward. The amount a commodity borrower is willing to pay must equal the amount the lender requires in return for lending out the commodity for time *T*. This interest or lease amount is an important factor in establishing the forward price for the commodity.

The commodity forward price for time *T* with an active lease market is expressed as:

$$F_{0,T} = S_0 e^{(r - \delta_l)T}$$

where:
S_0 = commodity current spot rate
$r - \delta_l$ = risk-free rate less the lease rate

The lease rate, δ_l, is income earned only if the commodity is loaned out.

Example: Pricing a commodity forward with a lease payment

Calculate the 12-month forward rate for a bushel of corn that has a spot rate of $5 and an annual lease rate of 7%. The appropriate continuously compounding annual risk-free rate for the commodity is equivalent to 9%.

Answer:

We can determine the 12-month forward rate as follows:

$$F_{0,T} = (S_T)e^{(r - \delta_l)T} = \$5 \times e^{(0.09 - 0.07)} = \$5.101$$

To further illustrate that this relationship must hold, consider the following no-arbitrage example.

Example: No-arbitrage for a commodity forward

Assume there is an active lending market for a bushel of corn. If no-arbitrage positions exist, **calculate** the forward price of a bushel of corn in one year if the lease rate is equal to 9%, the effective annual risk-free rate is equal to 9%, and the expected spot rate in one year is equal to $2/bushel of corn.

Answer:

Figure 1 represents a no-arbitrage opportunity for a bushel of corn. An investor could borrow money at the risk-free rate of 9% to purchase a bushel of corn and short sell it forward. The investor immediately lends the bushel of corn out at a lease rate of 9%. At the end of the lease period, T_1, the individual would pay back the loan with interest at $2.18, sell the corn at $2.00, and receive the lease payment of $0.18. In order for a no-arbitrage position to exist, the forward rate, $F_{0,1}$, must be equal to the expected spot rate of $2.00.

Figure 1: No-Arbitrage Opportunity on Bushel of Corn

Transaction	*Time* = T_0	*Time* = T_1
Borrow @ 9%	$2.00	$(2.18)
Buy a bushel of corn	$(2.00)	$2.00
Lend bushel of corn	$0	$0.18
Short forward @ $2	$0	$F_{0,1}$–$2
Total	$0	$F_{0,1}$–$2

Contango and Backwardation

An upward-sloping forward curve indicates that forward prices more distant in time are higher than current forward prices. The market is described as being in **contango** with an upward-sloping forward curve. A contango commodity market occurs when the lease rate is less than the risk-free rate. Based on the commodity forward formula, $F_{0,T} = S_0e^{(r - \delta_1)T}$, if $r > \delta_1$, the forward rate must be greater than the spot rate.

The market is described as being in **backwardation** with a downward-sloping forward curve. A backwardation commodity market occurs when the lease rate is greater than the risk-free rate. Based on the commodity forward formula, $F_{0,T} = S_0e^{(r - \delta_1)T}$, if $r < \delta_1$, the forward rate must be less than the spot rate.

Storage Costs

AIM 33.8: Define carry markets and explain the impact storage costs and convenience yields have on commodity forward prices and no-arbitrage bounds.

AIM 33.9: Compute the forward price of a commodity with storage costs.

When holding a commodity requires storage costs, *the forward price must be greater than the spot price* to compensate for the physical storage costs (i.e., costs associated with constructing and maintaining a storage facility) and financial storage costs (i.e., interest). The owner of a commodity can either sell it today for a price of S_0 or for delivery at time T at the forward price. If the owner sells it at a forward price, this is known as *cash-and-carry* (as we saw in AIM 33.6) because the seller receives the cash but must store (i.e., carry) the commodity until the delivery date. The market in which a commodity is stored is referred to as a **carry market.** The owner will only store the commodity if the forward price is greater than or equal to the expected spot price plus storage costs. This is represented mathematically as:

$$F_{0T} \geq S_0e^{rT} + \lambda(0,T)$$

where:
$\lambda(0,T)$ = FV of storage costs for one unit of the commodity from time 0 to T

If *storage costs are paid continuously and are proportional to the value of the commodity*, the no-arbitrage forward price becomes:

$$F_{0,T} = S_0e^{(r + \lambda)T}$$

where:
λ = continuous annual storage cost proportional to the value of the commodity

Example: Commodity forward pricing with storage costs and effective interest

Calculate the 3-month forward price for a bushel of soybeans if the current spot price is $3/bushel, the effective monthly interest rate is 1%, and the monthly storage costs are $0.04/bushel.

Answer:

First, calculate the future cost of storage for three months, $\lambda(0,T)$, as follows:

$$\$0.04 + \$0.04(1.01) + \$0.04(1.01)^2 = \$0.1212$$

The amount of $0.1212 represents the three months storage costs plus interest. Next, add the cost of storage to the spot price plus interest.

$$F_{0,T} = S_0e^{rT} + \lambda(0,T) \approx \$3.00(1.01^3) + \$0.1212 = \$3.0909 + \$0.1212 = \$3.2121$$

Professor's Note: Notice the approximation used in the previous example: $F_{0,T} = S_0e^{rT} \approx S_0 \times (1 + r)^T$. *Using either approach will produce similar results.*

Convenience Yield

If the owners of the commodity need the commodity for their business, holding physical inventory of the commodity creates value. For example, assume a manufacturer requires a specific commodity as a raw material. To reduce the risk of running out of inventory and slowing down production, excess inventory is held by the manufacturer. This reduces the risk of idle machines and workers. In the event that the excess inventory is not needed, it can always be sold. Holding an excess amount of a commodity for a non-monetary return is referred to as **convenience yield.**

A convenience yield *cannot* be earned by the average investor who does not have a business reason for holding the commodity. The forward price including a convenience yield is calculated as follows:

$$F_{0,T} \geq S_0e^{(r + \lambda - c)T}$$

where:
c = continuously compounded convenience yield, proportional to the value of the commodity

For the investor who does not earn the convenience yield, cash-and-carry arbitrage implies that:

$$F_{0,T} \leq S_0e^{(r + \lambda)T}$$

Example: Impact of convenience yield on the no-arbitrage cash-and-carry commodity forward pricing range

Suppose the owner of a commodity decides to lend out the commodity. The commodity has a continuously compounded convenience yield of *c*, proportional to the value of the commodity. **Determine** which range of prices must represent the no-arbitrage cash-and-carry opportunity for an investor who recognizes a convenience yield.

Answer:

The owner of a commodity is able to create a range of no-arbitrage prices as follows:

$$S_0e^{(r + \lambda - c)T} \leq F_{0,T} \leq S_0e^{(r + \lambda)T}$$

The upper bound depends on storage costs but not on the convenience yield. The lower bound adjusts for the convenience yield and therefore explains why forward prices may appear lower at times when the convenience yield is accounted for.

Comparing Lease Rates, Storage Costs, and Convenience Yield

AIM 33.10: Compare the lease rate with the convenience yield.

Here is a handy guide for relating forward and spot commodity prices on the exam. Start with the basic expression relating forward and spot prices:

$$F_{0,T} = S_0 e^{rT}$$

This expression says that if there are no costs or benefits associated with buying and holding the commodity, the forward price is just the spot price compounded at the risk-free rate over the holding period.

If there are benefits (e.g., lease rates, convenience yield) to buying the commodity today, the holder is willing to accept a lower forward price. The forward price is reduced by the benefit, either the lease rate or convenience yield:

$$F_{0,T} = S_0 e^{(r - c)T} < S_0 e^{rT}$$

where c = the convenience yield, or

$$F_{0,T} = S_0 e^{(r - \delta)T} < S_0 e^{rT}$$

where δ = the lease rate

If there are costs, such as storage costs, associated with purchasing the commodity today, the forward price is increased by the cost:

$$F_{0,T} = S_0 e^{(r + \lambda)T} > S_0 e^{rT}$$

where λ = the storage costs

Of course, there can be combinations of costs and benefits, so be sure to increase the exponent for costs and reduce it for benefits:

$$F_{0,T} = S_0 e^{(r + \lambda - c)T}$$

In the equation above, the lease rate is equal to storage costs minus the convenience yield.

Commodity Characteristics

AIM 33.11: Discuss factors that impact gold, corn, natural gas, and crude oil futures prices.

Certain commodities exhibit unique properties that impact their forward price. For example, gold, corn, natural gas, and oil are all commodities with characteristics that differ with respect to storage costs, the ability to store, production costs, and seasonal demand.

These differences are reflected in lease rates, storage costs, and convenience yields that influence the commodity forward prices and the shape of the forward curves.

Gold Forward Price Factors

Because gold can earn a return by being loaned out, strategies for holding synthetic gold offer a higher return than holding just the physical gold without lending it out. When a positive lease rate is present, the synthetic gold is preferred to physically holding the gold because the lease rate represents the cost of holding the gold without lending it.

The value of gold is also influenced by the cost of production. The present value of gold received in the future is simply the present value of the forward price computed at the risk-free rate of return. The present value of gold production is calculated as follows:

$$\text{PV of gold production} = \sum_{i=1}^{n} n_{t_i}\left[F_{0,t_i} - x(t_i)\right] e^{-r(0,t_i)t_i}$$

where:
n_{t_i} = amount of ounces of gold we expect to extract, with an extraction cost of $x(t_i)$

Under this framework, the gold mine is assumed to operate the entire time, and production is known with certainty.

Corn Forward Price Factors

Corn is an example of a commodity with seasonal production and a constant demand. Corn is produced every fall, but it is consumed throughout the year. In order to meet consumption needs, corn must be stored. Thus, interest and storage costs need to be considered. The price of the corn will fall as it is being harvested and then rise to reflect the cost of storage over the next 12 months until it is harvested again. Thus, the forward curve is increasing until harvest time, and it drops sharply and slopes upward again after harvest time is over.

Example: Corn commodity pricing with storage costs

Suppose the spot price today for a bushel of corn is \$2.25, the continuously compounded interest rate is 5.5%, and the storage cost is 2.0% per month. **Calculate** the 6-month forward price.

Answer:

$$F_{0,0.5} = \$2.25 \times e^{(0.00458 + 0.02)6} = \$2.25 \times 1.15893 = \$2.61$$

Professor's Note: The 0.458% used for the monthly interest rate is the annual rate divided by 12.

Natural Gas Forward Price Factors

Natural gas is an example of a commodity with constant production but seasonal demand. Natural gas is expensive to store, and demand in the United States peaks during high periods of use in the winter months. In addition, the price of natural gas is different for various regions due to high international transportation costs. Storage is at its peak in the fall just prior to the peak demand. Therefore, the forward curve rises steadily in the fall.

Example: Calculation of natural gas forward price with storage costs

Calculate the natural gas implied storage cost for the month of October if the October 2005 spot price is 4.071, the annual risk-free rate of interest is 6%, and the November forward price is 4.157.

Answer:

$$\$4.157 = \$4.071e^{0.005} + \lambda_{Oct2005}$$

$$\$4.157 = \$4.091 + \lambda_{Oct2005}$$

$$\$4.157 - \$4.091 = \lambda_{Oct2005}$$

$$\$0.066 = \lambda_{Oct2005}$$

Oil Forward Price Factors

The physical characteristics of oil make it is easier to transport than natural gas. Therefore, the price of oil is comparable worldwide. In addition, demand is high in one hemisphere when it is low in the other. Lower transportation costs and more constant worldwide demand causes the long-run forward price to be more stable. In the short-run, supply and demand shocks cause more volatile prices because supply is fixed. For example, the Organization of Petroleum Exporting Countries (OPEC) may decrease supply to increase prices by causing a shortage in the short run. Supply and demand adjust to price changes in the long run.

Commodity Spread

AIM 33.12: Define and compute a commodity spread.

A **commodity spread** results from a commodity that is an input in the production process of other commodities. For example, soybeans are used in the production of soybean meal and soybean oil. A trader creates a **crush spread** by holding a long (short) position in soybeans and a short (long) position in soybean meal and soybean oil.

Similarly, oil can be refined to produce different types of petroleum products such as heating oil, kerosene, or gasoline. This process is known as "cracking," and thus the difference in prices of crude oil, heating oil, and gasoline is known as a **crack spread**. For example, seven gallons of crude oil may be used to produce four gallons of gasoline and

three gallons of heating oil. Commodity traders refer to the crack spread as 7-4-3, reflecting the seven gallons of crude oil, four gallons of gasoline, and three gallons of heating oil. Thus, an oil refiner could lock in the price of the crude oil input and the finished good outputs by an appropriate crack spread reflecting the refining process. However, this is not a perfect hedge because there are other outputs that can be produced such as jet fuel and kerosene.

Example: Pricing a crack (commodity) spread

Suppose we plan on buying crude oil in one month to produce gasoline and kerosene for sale in two months. The 1-month futures price for crude oil is currently \$30/barrel. The 2-month future prices for gasoline and heating oil are \$41/barrel and \$31.50/barrel, respectively. **Calculate** the 5-3-2 crack (commodity) spread.

Answer:

The 5-3-2 spread tells us the amount of profit that can be locked in by buying five barrels of oil and producing three barrels of gasoline and two barrels of heating oil.

profit for a 5-3-2 spread =
(3 × \$41) + (2 × \$31.50) – (5 × \$30) = \$123 + \$63 – \$150 = \$36 for five barrels, or
\$36 / 5 barrels = \$7.20/barrel

Professor's Note: There is no calculation for interest adjustment in this example.

Basis Risk

AIM 33.13: Explain how basis risk can occur when hedging commodity price exposure.

As you may recall, **basis** is the difference between the spot price (or rate) and the price (or rate) of the futures contract used to hedge. If the values of both move together perfectly, an investor long or short the asset can lock in a return or value by selling or buying futures, respectively.

Professor's Note: When you expect to receive the commodity in the future, we say you are long the commodity and you will hedge the value of the expected commodity by selling the corresponding futures contracts. If you will deliver the commodity in the future, you are short, and you will hedge by taking a long position in the corresponding futures contracts.

Any time the values of the spot and futures contracts do not move together perfectly, the hedger faces **basis risk**. An example with financial futures is using a basket currency futures contract to hedge the value of a transaction in an emerging market. Since the hedged asset (i.e., the emerging market currency) and the underlying in the futures contract are not identical, there is risk associated with changes in their relative values. Also, if the financial

futures contract must be rolled over, or if it matures after the delivery date, this adds to the basis risk.

Since there are storage and transportation costs associated with commodities, hedgers face more concerns. As with financial futures, every commodity futures contract specifies a delivery amount and a delivery date. In addition, however, every commodity futures contract specifies a delivery *location* and the deliverable *grade* (i.e., quality). For example, an investor planning to receive oil in New York City might use NYMEX futures, which specify delivery in Oklahoma. At the producer level, an Iowa corn farmer might use CBOT corn futures, which specify delivery in Chicago.

Strip Hedge vs. Stack Hedge

AIM 33.14: Evaluate the differences between a strip hedge and a stack hedge and analyze how these differences impact risk management.

An oil producer may enter into a contract to supply a fixed amount of barrels of oil per month at a fixed price. The oil producer could set up a **strip hedge** by buying futures contracts that match the maturity and quantity for every month of the obligation.

To help reduce transaction costs, the oil producer might instead utilize a **stack hedge**. To form a stack hedge, the oil producer would enter into a one-month futures contract equaling the total value of the year's promised deliveries. As transactions costs are less for short-term (e.g., one-month) contracts, the total cost of implementing this strategy is less than for a comparable strip hedge. At the end of the first month, the producer rolls into the next one-month contract, and so forth, each month setting the total amount of the contract equal to the remaining promised deliveries. This strategy of continually rolling into the next near-term contract is referred to as **stack and roll**.

A stack hedge has the advantage when near-term contracts are more readily available due to heavier volume and more liquidity. Another advantage of near-term contracts is that distant futures on commodities often have wider bid-ask spreads and therefore larger transaction costs. In addition, an oil producer may prefer a stack hedge in order to speculate on the shape of the forward curve. For example, assume the forward curve looks unusually steep. The oil producer would then enter into a stacked hedge with a large near-term contract. If the forward curve later flattens, the oil producer locks in all the oil at a relatively cheap near-term price compared to the more expensive futures using the strip strategy.

Example: Creation of a strip or stack hedge

Determine how an oil producer could hedge the risk of an agreement to supply 150,000 barrels of oil each month for a year at a fixed price.

Answer:

The oil producer could enter into a strip hedge by obtaining a long futures contract position for every month of the year for 150,000 barrels.

Alternatively, the oil producer could create a long position of a near-term futures contract for a little less than 1,800,000 barrels. At the end of the month, the oil producer would enter into a new near-term futures contract for a smaller amount representing the present value of future deliveries.

Cross Hedging

AIM 33.15: Describe examples of cross-hedging, specifically hedging jet fuel with crude oil and using weather derivatives.

In some cases, a futures contract with an underlying instrument that is exactly the same as the position to be hedged will not exist. For example, there are no contracts for jet fuel futures in the United States. Therefore, hedging jet fuel requires a **cross hedge**. Some firms hedge the cost of jet fuel with crude oil futures while others hedge using a combination of crude oil and heating oil futures. Three factors are relevant when making a cross hedge decision:

- The liquidity of the futures contract (since delivery may not be an option).
- The correlation between the underlying for the futures contract and the asset(s) being hedged.
- The maturity of the futures contract.

Each of these factors has an impact on the effectiveness of the hedge. The liquidity of the cross hedge is important in order for the portfolio manager to quickly unwind the futures obligation. Thus, the manager should try to choose among liquid instruments to find the futures contract whose maturity most closely matches that of the horizon of the hedged position.

To illustrate the concept of cross hedging, consider a firm that uses crude oil futures to hedge jet fuel prices. The payoff from this type of hedge will depend on both the change in jet fuel prices and the change in oil futures prices. Thus, the number of crude oil futures contracts required is estimated using regression analysis, where the change in jet fuel prices is dependent on the change in oil futures prices. The slope coefficient from the regression results will provide the portfolio manager with hedge ratio information regarding the degree that crude oil price changes affect the price of jet fuel.

A cross hedge is also applied when firms use **weather derivatives**. Weather risk is a business risk that is faced by agricultural firms as well as many firms involved with providing recreational services. It refers to any financial losses, explicit and implicit, that a firm faces from changes in the weather.

Utility companies use weather derivatives, which are based on "degree days," to hedge the cost of energy purchases. Much of the energy supplied by utilities is used for heating or cooling with variations in demand directly correlated with weather patterns. Demand can rise and fall dramatically in conjunction with the weather experienced in the areas that the utilities service.

Utilities can use derivatives with payoffs based on the weather experienced at weather stations that are representative of the areas that they serve. For example, a utility located in the northeast U.S. contracts for energy needs based on average weather experienced over previous years and predictions for the coming year. Unhedged, the utility would leave itself exposed to rising prices from energy producers in the event that the coming winter is far worse than predicted.

If hedging with weather derivatives (specifically weather options), and the winter were worse than expected (have more heating degree days than the strike value of the contract), the utility would receive the specified payment. If the winter were milder than expected, the contract would expire worthless. The actual measurements are from specified U.S. government sites in the areas specified by the contract.

The use of weather derivatives by other investors is growing, but one of the biggest problems is basis risk. That is, it is difficult to accurately match up the exposure of other assets to the weather with that specified by the contracts. Other than large-scale exposure, such as that experienced by utilities, many producers are much more susceptible to more local variations. For instance, a large farming operation has exposure to the rain falling on its own fields and may suffer losses from too much or too little rain. The rain on its fields may not have a high correlation with the rain experienced at the weather station 50 miles away.

Key Concepts

1. The commodity forward price today is defined as a biased estimate of the expected spot commodity price at time *T* as follows: $F_{0,T} = E(S_T)e^{(r-\alpha)T}$.

2. The lease rate is defined as the amount of return the investor requires to buy and then lend a commodity. If an active lease market exists for a commodity, a commodity lender can earn the lease rate by buying a commodity and immediately selling it forward.

3. The commodity market is in contango with an upward-sloping forward curve when the lease rate is less than the risk-free rate. The market is in backwardation with a downward-sloping forward curve when the lease rate is greater than the risk-free rate.

4. A commodity owner will only store the commodity if the forward price is greater than or equal to the spot price plus the future storage costs as follows: $F_{0,T} \geq S_0e^{rT} + \lambda(0,T)$, where $\lambda(0,T)$ represents the future value of storage costs for one unit of the commodity from time 0 to *T*.

5. If storage costs are paid continuously and are proportional to the value of the commodity, the no-arbitrage forward price becomes $F_{0,T} = S_0e^{(r+\lambda)T}$.

6. Holding an excess physical inventory of the commodity creates non-monetary value for commodity owners who require the commodity as a production input. This is referred to as convenience yield, and the forward price including a convenience yield is calculated as $F_{0,T} \geq S_0e^{(r+\lambda-c)T}$, where *c* is the continuously compounded convenience yield, proportional to the value of the commodity.

7. The owner of a commodity who uses the commodity in production is able to create a range of no-arbitrage prices as follows: $S_0e^{(r+\lambda-c)T} \leq F_{0,T} \leq S_0e^{(r+\lambda)T}$.

8. Gold, corn, natural gas, and oil are all examples of commodities with characteristics that differ with respect to storage costs, the ability to store, production costs, and seasonal demand. These unique differences influence the commodity forward prices and the shape of the forward curves.

9. A commodity spread results from a commodity that is an input in the production process of other commodities. For example, a 7-4-3 crack spread refers to the profit for holding four gasoline futures plus three heating oil futures less seven crude oil futures.

10. Basis risk results from the inability to create a perfect hedge due to differences in the commodities with respect to timing, grade, storage costs, and/or transportation costs.

11. A strip hedge is created by buying future contracts that match the maturity and quantity for every month of the obligation. A stack hedge is created by buying a futures contract with a single maturity based on the present value of the future obligations. Advantages of the stack hedge are the availability and liquidity of near-term contracts and narrower bid-ask spreads for near-term contracts.

Concept Checkers

1. Which of the following statements regarding lease rates is(are) true? The lease rate is:

   I. the amount of return the investor requires to buy and then lend a commodity.
   II. very similar to the dividend payment in a financial forward.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

2. Suppose there is an active lending market for a bushel of soybeans. If the annual lease rate is equal to 7%, the effective annual risk-free rate is equal to 7%, and the expected spot rate in one year is equal to $4/bushel of soybeans, how could an investor create an arbitrage opportunity? An individual could:
   A. borrow money at 7% and purchase a bushel of soybeans and sell it forward.
   B. borrow a bushel of soybeans and sell a bushel of soybeans at the spot rate and buy a long forward.
   C. sell a bushel of soybeans at the forward rate and lend the money at the risk-free rate.
   D. go long in soybean forward contracts, short in soybean spot rates, and lend the excess proceeds at the risk-free rate.

3. What is the 3-month forward price for a bushel of corn if the current spot price for corn is $3/bushel, the effective monthly interest rate is 1.5%, and the monthly storage costs are $0.03/bushel?
   A. $3.18.
   B. $3.23.
   C. $3.29.
   D. $3.31.

4. Suppose we plan on buying crude oil in one month to produce gasoline and heating oil for sale in two months. The 1-month future price for crude oil is currently $42.5/barrel. The 2-month future prices for gasoline and heating oil are $45/barrel and $43.50/barrel, respectively. What is the 7-5-2 crack (commodity) spread?
   A. $2.07/barrel.
   B. $6.00/barrel.
   C. $14.50/barrel.
   D. $22.09/barrel.

5. Which of the following statements is an example of basis risk? Purchasing:
   A. an oil contract with delivery in a different geographical region.
   B. a commodity with a desired distant delivery with near-term contracts.
   C. a eurodollar contract, due to lack of commodity futures.
   D. All of the above statements are correct.

CONCEPT CHECKER ANSWERS

1. C A *lease rate* is the amount of interest a lender of a commodity requires. From the borrower's perspective, the lease rate represents the cost of borrowing the commodity. The lease rate in the pricing of a commodity future is very similar to the dividend payment in a financial forward.

2. A An individual could borrow money at the risk-free rate of 7% to purchase a bushel of soybeans and sell it forward. The individual immediately lends the bushel of soybeans out at a lease rate of 7%. At the end of the lease period, T_1, the individual would pay back the loan with interest at \$4.28, sell the soybeans at \$4.00, and receive the lease payment of \$0.28. In order for a no-arbitrage position to exist, the forward rate, $F_{0,1}$, must be equal to the expected spot rate of \$4.00. An arbitrage position exists if the forward rate is not equivalent to the expected spot rate.

No-Arbitrage Opportunity on Bushel of Soybeans

Transaction	*Time* = T_0	*Time* = T_1
Borrow @ 7%	\$4.00	(\$4.28)
Buy a bushel of soybeans	(\$4.00)	\$4.00
Lend bushel of soybeans	\$0	\$0.28
Short forward @ \$4	\$0	$F_{0,1}$–\$4
Total	\$0	$F_{0,1}$–\$4

3. B First calculate the future cost of storage for three months, $\lambda(0,T)$, as follows:

$$\$0.03 + \$0.03(1.015) + \$0.03(1.015)^2 = \$0.0914$$

The amount of \$0.0914 represents the 3-month storage costs plus interest. Next, add the cost of storage to the spot price plus interest.

$$F_{0,T} = S_0e^{rT} + \lambda(0,T) \approx \$3.00(1.015^3) + \$0.0914 = \$3.1370 + \$0.0914 = \$3.23$$

4. A The 7-5-2 spread tells us the amount of profit that can be locked in by buying seven barrels of oil and producing five barrels of gasoline and two barrels of heating oil.

Profit for a 7-5-2 spread =

(5 × \$45) + (2 × \$43.50) – (7 × \$42.5) = \$225 + \$87 – \$297.5 = \$14.50 for seven barrels, or \$14.5 / 7 barrels = \$2.07/barrel.

5. D All are examples of basis risk, which results from the inability of commodities to create a perfect hedge. Differences due to timing, grade, storage costs, or transportation costs create basis risk.

The following is a review of the Financial Markets and Products principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Foreign Exchange Risk

Topic 34

Exam Focus

Exposure to foreign exchange risks is a natural result of the globalization of financial institutions. These risks arise when foreign currency trading and/or foreign asset-liability positions are mismatched in individual currencies. Unexpected volatility can generate significant losses for the firm, which could, in turn, threaten profitability or even solvency. These risks can be mitigated by direct hedging through matching foreign asset-liability books of business, hedging through forward contracts, and through foreign asset and liability portfolio diversification.

Sources of Foreign Exchange Risk

AIM 34.1: Calculate a financial institution's overall foreign exchange exposure.

AIM 34.2: Demonstrate how a financial institution could alter its net position exposure to reduce foreign exchange risk.

AIM 34.3: Calculate a financial institution's potential dollar gain or loss exposure to a particular currency.

Large financial institutions (banks) frequently take significant positions in foreign currency assets and liabilities as a result of their foreign exchange trading activities. When looking at such financial institutions' currency trading activities, the aggregate position size in a particular currency may look extremely large; however, since buys and sells will offset one another in terms of exposure, the net exposure to the currency may actually be quite small.

A bank's actual exposure to any given currency can be measured by the **net position exposure.** Net exposure is the extent to which a bank is net long (or *positive*) or net short (or *negative*) in a given currency. For example, a bank's net euro (EUR) exposure would be:

$$\text{net EUR exposure} = (\text{EUR assets} - \text{EUR liabilities}) + (\text{EUR bought} - \text{EUR sold})$$

$$\text{net EUR exposure} = \text{net EUR assets} + \text{net EUR bought}$$

A **positive net exposure** position means that we are *net long in a currency.* In other words, we hold more assets than liabilities in a given currency. In this instance, the financial institution faces the risk that the foreign currency will *fall* in value against the domestic currency.

A **negative net exposure** position means that we are *net short in a currency.* The financial institution faces the risk that the foreign currency will *rise* in value against the domestic currency.

Therefore, if a U.S. financial institution fails to maintain a balanced position in a currency where assets (purchases) are exactly offset by liabilities (sales), the institution will be exposed to variations in the foreign exchange (FX) rate of that currency against the U.S. dollar. The more volatile the FX rate, the more potential impact a net exposure (either long or short) will have on the value of a bank's foreign currency portfolio.

Foreign Trading Activities

AIM 34.4: List and describe the different types of foreign exchange trading activities.

A financial institution's buying and selling of foreign currencies, and hence the institution's position in the FX market, reflects four key trading activities:

1. Enabling customers to participate in international commercial business transactions.
2. Enabling customers to take positions in real or financial foreign investments. Note that a financial institution may also transact in foreign currencies to take positions in real or financial foreign investments for its own portfolio.
3. Offsetting exposure in a given currency for hedging purposes.
4. Speculating on foreign currencies in search of profit by forecasting and/or anticipating futures FX rate movements.

When a bank is buying or selling a foreign currency for the purpose of either allowing its customers to participate in international commercial business transactions or investing in real or financial foreign investments, the bank typically serves as an agent for the customers (receives a fee) and does not assume the FX risk itself.

When a bank is buying or selling a currency for hedging purposes, this will reduce FX exposure.

The fourth activity, trading foreign currencies with the intent to profit by anticipating future foreign currency rate movements, relates to open positions that are taken for speculative purposes and represents an unhedged position in a given currency. These speculative trades are usually made directly with other financial institutions or arranged through FX specialist brokers.

Currency spot trades are the most frequently executed speculative trades. The financial institution seeks to earn a profit on the difference between the buy and sell prices or on movements in the bid-ask spreads over time. Speculative positions can also be taken in FX forward contracts, futures, and options.

Sources of Profits and Losses on Foreign Exchange Trading

AIM 34.5: Identify the sources of foreign exchange trading gains and losses.

AIM 34.6: Calculate the potential gain or loss from a foreign currency denominated investment.

Most returns on FX trading arise from speculation in currencies or taking an unhedged position in a particular currency. Financial institutions also earn fees as a secondary source of revenues. These revenues are earned from market-making activities and/or from acting as agents for retail or wholesale customers.

Mismatched Foreign Asset and Liability Positions

A financial institution can also have foreign exchange exposure due to mismatches between foreign financial asset and liability portfolios. The following example shows the exposure resulting from such a mismatch.

Example: Foreign investment returns

Figure 1: Balance Sheet

Assets	*Liabilities*
USD50 million U.S. loans, 1-year maturity, in USD, yielding 8%	USD100 million U.S. CDs, 1-year maturity, in USD, yielding 6%
USD50 million equivalent Swiss loans, 1-year maturity, made in CHF, yielding 13%	

This firm has matched the duration of its assets and liabilities ($D_A = D_L = 1$ year) but has mismatched the currency composition of its portfolio. Note that the firm would earn a positive spread of 2% (8% – 6%) from investing domestically. In order to invest in Switzerland, this firm decides to take 50% of its $100 million and make 1-year Swiss loans while keeping 50% to make U.S. dollar loans. What transactions must the firm undertake to make the CHF-denominated loan (assuming the FX position is not hedged)?

Answer:

1. Sell USD50 million for CHF on the spot currency markets at the beginning of the year. If the exchange rate is USD1.70 to 1 CHF, this yields USD50,000,000 / 1.7 = CHF29,411,765.
2. Use the CHF29,411,765 to make 1-year Swiss loans at a 13% interest rate.
3. At the end of the one year, CHF revenue from these loans will be CHF29,411,765(1.13) = CHF33,235,294 (assuming no default).
4. At the end of the year, repatriate these funds back to the United States. In other words, the U.S. bank will sell CHF33,235,294 in the FX market at the spot exchange rate that exists at the end of the year.

In this example, we assume the spot FX rate has not changed over the 1-year period and remains at USD1.70/CHF. The dollar proceeds from the Swiss investment would be:

$$\text{CHF}33{,}235{,}294 \times \text{USD}1.70\,/\,\text{CHF} = \text{USD}56{,}500{,}000, \text{ for a return of:}$$

$$\frac{\text{USD}56{,}500{,}000 - \text{USD}50{,}000{,}000}{\text{USD}50{,}000{,}000} = 13.0\%$$

Thus, the weighted return on this portfolio will be:

$$(0.5)(0.08) + (0.5)(0.13) = 0.105 \text{ or } 10.5\%$$

This exceeds the cost of the bank CDs by 4.5% (=10.5% – 6.0%).

Example, continued:

Now, suppose that at the end of the year, the Swiss franc has *fallen* in value relative to the U.S. dollar. If the exchange rate is now USD1.55/CHF, **compute** what the Swiss loan revenues would be at the end of Year 1.

Answer:

The Swiss loan revenues at the end of one year equal:

$$\text{CHF}33{,}235{,}294 \times \text{USD}1.55\,/\,\text{CHF} = \text{USD}51{,}514{,}706, \text{ for a return of:}$$

$$\frac{\text{USD}51{,}514{,}706 - \text{USD}50{,}000{,}000}{\text{USD}50{,}000{,}000} = 3.03\%$$

Thus, the weighted return on this portfolio will be:

$$(0.5)(0.08) + (0.5)(0.0303) = 0.0552 \text{ or } 5.52\%$$

Under this scenario, the bank would actually have a negative interest margin on its balance sheet investments of –0.48% since its cost of funds (COFs) is 6.0%.

Example, continued:

If the Swiss franc had *appreciated* against the dollar over the year, the bank would have generated a double benefit: (1) from the appreciation of the franc, and (2) from the higher yield on the domestic Swiss loans. If the exchange rate is now USD1.82/CHF, **compute** what the Swiss loan revenues would be at the end of Year 1.

Answer:

$$\text{CHF}33{,}235{,}294 \times \text{USD}1.82\,/\,\text{USD} = \text{USD}60{,}488{,}235, \text{ for a return of:}$$

$$\frac{\text{USD}60{,}488{,}235 - \text{USD}50{,}000{,}000}{\text{USD}50{,}000{,}000} = 20.98\%$$

The previous example illustrates an important concept. As with any investment, returns for the bank's portfolio are derived from differences between income and costs. However, foreign investing provides the additional dynamic of having profits or losses affected by changes in foreign exchange rates. There are two principle methods available to control the scale of FX exposure: on-balance-sheet hedging and off-balance-sheet hedging.

On-Balance-Sheet Hedging

AIM 34.7: Explain balance-sheet hedging with forwards.

On-balance-sheet hedging is achieved when a financial institution has a matched maturity and currency foreign asset-liability book. Figure 2 is an illustration.

Figure 2: Balance Sheet

Assets	*Liabilities*
USD50 million U.S. loans, 1-year maturity, in USD, yield 8%	USD50 million U.S. CDs, 1-year maturity, in USD, yielding 6%
USD50 million equivalent Swiss loans, 1-year maturity, made in CHF, yielding 13%	USD50 million Swiss CDs, 1-year maturity, raised in CHF, yielding 10%

Using the data in Figure 2, we can examine the effects of the franc depreciating by the same amount as in the previous example:

1. The bank borrows USD50 million equivalent in Swiss francs for one year at an interest rate of 10%. At the exchange rate of USD1.70/CHF, this equates to USD50,000,000 / 1.70 = CHF29,411,765.
2. At the end of one year, the bank must pay back the Swiss franc CD holders their principal and interest: CHF29,411,765 × (1.10) = CHF32,352,941.
3. If the franc *depreciated* to USD1.55/CHF in the period, repayment in dollar terms would be CHF32,352,941 × USD1.55/CHF = USD 50,147,059, or a dollar cost of funds of 0.3%.
4. The bank makes CHF29,411,765 in loans at 13% for one year.
5. At the end of one year, the loans are repaid with interest. CHF29,411,765 (1.13) = CHF33,235,294, but at USD1.55/CHF, this equals only USD51,514,706 for a return of 3.03%.

At the end of the year, we would have the following.

Average return on assets:

$$(0.5)(0.08)+(0.5)(0.0303)=0.0552 \text{ or } 5.52\%$$
$$\text{U.S. asset return} + \text{CHF asset return} = \text{overall return}$$

Average cost of funds:

$$(0.5)(0.06)+(0.5)(0.003)=0.0315 \text{ or } 3.15\%$$
$$\text{U.S. cost of funds} + \text{CHF cost of funds} = \text{overall cost}$$

Net return:

$$5.52\% - 3.15\% = 2.37\%$$
$$\text{average return on assets} - \text{average cost of funds}$$

By directly matching foreign assets and liabilities, we can lock in a positive return or profit spread if exchange rates move in either direction over the investment period.

Off-Balance-Sheet Hedging

Rather than matching foreign assets with foreign liabilities, we may choose to remain unhedged on the balance sheet. If we do, we could hedge off-balance-sheet by taking a position in the forward market. This hedge would appear as a contingent off-balance-sheet claim as an item below the net income line.

Referring to the previous example, the function of the forward FX contract is to offset the uncertainty of the future spot rate on the CHF at the end of the investment horizon. A forward foreign exchange agreement involves the exchange of a foreign currency at some point in the future at an exchange rate that is determined today. Rather than repatriating CHF and exchanging them for USD at the end of the period at an unknown rate, the bank can enter into a contract to sell forward the *expected* principal and interest on the loan at the current known **forward exchange rate** for USD/CHF, with the delivery of Swiss francs to the buyer of the forward contract taking place at the end of the investment horizon. This method effectively removes the future spot exchange rate uncertainty that is related to investment returns on the Swiss loan. By using the data in Figure 2, we can illustrate how this technique would work.

Example: Hedging with forward contracts

Outline the transactions necessary for the financial institution to use an off-balance-sheet hedge for the asset-liability position described in Figure 2.

Answer:

The following transactions create the off-balance-sheet hedge.

1. The U.S. bank sells USD50 million for Swiss francs at the *spot* exchange rate *today* and receives USD50,000,000 / USD1.7/CHF = CHF29,411,765.

2. Immediately after the sale, the bank lends the CHF29,411,765 to a Swiss customer at 13% for one year.

3. In addition, the bank sells the expected principal and interest proceeds from the franc loan forward for U.S. dollars at today's forward rate (say, USD1.65/CHF) for 1-year delivery: (USD1.65 – USD1.70) / USD1.70 = –2.94%.

   The forward buyer of the francs will pay USD54,838,235 to the seller when the bank delivers the CHF33,235,294 proceeds of the loan to the financial institution seller.

   $$\begin{aligned}\text{CHF29,411,765}(1.13) \times \text{USD1.65/CHF} &= \text{CHF33,235,294} \times \text{USD1.65/CHF} \\ &= \text{USD54,838,235}\end{aligned}$$

4. At the end of one year, the Swiss borrower repays the loan to the bank plus interest in Swiss francs (CHF33,235,294).

5. The bank gives the CHF33,235,294 to the buyer of the 1-year forward contract and receives USD54,838,235.

By using this method, the bank knows it has locked in a guaranteed return of 9.68% on the Swiss franc (assuming, of course, the loan will not default and the forward buyer does not renege on the forward contract).

$$\frac{\text{USD54,838,235} - \text{USD50,000,000}}{\text{USD50,000,000}} = 0.0968 = 9.68\%$$

The overall expected return on the bank's asset portfolio would then be:

$$(0.5)(0.08) + (0.5)(0.0968) = 8.84\%$$

Regardless of spot exchange rate fluctuations over the year, the bank has locked in a risk-free return spread of 2.84% (8.84% return – 6% cost of funds) over the cost of funds for the bank's CDs.

AIM 34.8: Describe how a non-arbitrage assumption in the foreign exchange markets leads to the interest rate parity theorem; use this theorem to calculate forward foreign exchange rates.

Because the hedged Swiss loans offer a higher return than the U.S. loans, it makes sense for the bank to focus its activities on making hedged Swiss loans. However, as more is invested in Swiss loans, the bank must buy more Swiss francs. This will continually reduce the forward rate spread until no additional profits could be made by making the forward contract-hedged investments.

As the bank moves into more Swiss loans, the spot exchange rate for buying francs will rise. In equilibrium, the forward exchange rate would have to fall to completely eliminate the attractiveness of the Swiss investments.

This relationship is called **interest rate parity** (IRP) since the discounted spread between domestic and foreign interest rates equals the percentage spread between forward and spot exchange rates. In other words, the hedged dollar return on foreign investments should be equal to the return on domestic investments. IRP implies that in a competitive market, a firm should not be able to make excess profits from foreign investments (i.e., a higher domestic currency return from lending in a foreign currency and locking in the forward rate of exchange).

For the exam, you should know that the exact IRP equation using direct quotes is:

$$\text{forward} = \text{spot}\left[\frac{(1+r_{DC})}{(1+r_{FC})}\right]^T$$

where:
r_{DC} = domestic currency rate
r_{FC} = foreign currency rate

If this equality does not hold, an arbitrage opportunity exists. To remember this formula, note that when the forward and spot rates are expressed as direct quotes (DC/FC), right-hand side of the equation also has the domestic (interest rate) in the numerator and the foreign (interest rate) in the denominator.

If we expressed the forward and spot rates as indirect quotes (FC/DC), then the right-hand side of the equation would have the foreign (interest rate) in the numerator and the domestic (interest rate) in the denominator. So it's either domestic over foreign for everything, or foreign over domestic for everything.

IRP can also be stated using continuously compounded rates as follows:

$$\text{forward} = \text{spot} \times e^{(r_{DC}-r_{FC})T}$$

Example: Interest rate parity

Suppose you can invest in NZD at 5.127%, or you can invest in Swiss francs at 5.5%. You are a resident of New Zealand, and the current spot rate is 0.79005 NZD/CHF. Calculate the 1-year forward rate expressed in NZD/CHF.

Answer:

$$\text{forward}(DC/FC) = \text{spot}(DC/FC)\left[\frac{(1+r_{DC})}{(1+r_{FC})}\right] = 0.79005\left(\frac{1.05127}{1.055}\right) = 0.78726$$

Professor's Note: Notice here that the NZD/CHF rate fell from 0.79005 to 0.78726. This implies that it now takes fewer NZD to buy one CHF. So, in other words, the New Zealand dollar has appreciated relative to the Swiss franc. Consequently, the Swiss franc has depreciated relative to the New Zealand dollar.

Diversification in Multicurrency Foreign Asset-Liability Positions

AIM 34.9: Explain why diversification in multicurrency asset-liability positions could reduce portfolio risk.

AIM 34.10: Describe the relationship between nominal and real interest rates.

Our previous examples have used matched and mismatched asset-liability portfolios that involve only one foreign currency. In reality, most financial institutions hold positions in many different currencies in their asset-liability portfolios. Since currencies may be less than perfectly correlated, diversification across several asset and liability markets can potentially reduce portfolio risk as well as the cost of funds. Domestic and foreign interest rates and stock returns generally do not move together perfectly over time. This means that the risks from mismatching one-currency positions may be offset by potential gains from asset-liability portfolio diversification.

Each domestic and foreign nominal interest rate consists of two components. The first component is the **real interest rate,** which reflects a given currency's real demand and supply for its funds. Differences in real interest rates will cause a flow of capital into those countries with the highest available *real* rates of interest. Therefore, there will be an increased demand for those currencies, and they will appreciate relative to the currencies of countries whose available real rate of return is low.

The second component is the **expected inflation rate,** which reflects the amount of compensation required by investors to offset the expected erosion of real value over time due to inflation. Differences in inflation rates will cause the residents of the country with the highest inflation rate to demand more imported (cheaper) goods. For example, if prices in the United States are rising twice as fast as in Australia, U.S. citizens will increase their

demand for Australian goods (because Australian goods are now cheaper relative to domestic goods). If a country's inflation rate is higher than its trading partners', the demand for the country's currency will be low, and the currency will depreciate.

The **nominal interest rate,** *r*, is the compounded sum of the real interest rate, *real r*, and the expected rate of inflation, *E*(i), over an estimation horizon.

exact methodology: $(1 + \text{r}) = (1 + \text{real r})[1 + \text{E(i)}]$

linear approximation: $\text{r} \approx \text{real} + \text{E(i)}$

Key Concepts

1. Net exposure in a foreign currency measures the extent to which a bank is net long or net short a foreign currency. A net long (short) position in a currency means that a bank faces the risk that the FX rate will fall (rise) in value versus the domestic currency.

2. A financial institution's buying and selling of foreign currencies, and hence the institution's position in the FX market, reflects four key trading activities:
   - Enabling customers to participate in international commercial business transactions.
   - Enabling customers (or the financial institution itself) to take positions in real and financial foreign investments.
   - Offsetting exposure to gain currency for hedging purposes.
   - Speculating on future FX rate movements.

3. Most of the profits and losses on FX come from speculation or open position taking. A secondary source of revenue comes from market-making activities and/or agency fees.

4. Mismatches between foreign financial assets and liabilities can create FX risk exposure.

5. There is an extra dimension of return and risk from adding foreign currency assets and liabilities to a portfolio.

6. There are two principle methods of better controlling the impact of FX exposure:
   - On-balance-sheet hedging is achieved when a financial institution has a matched maturity and foreign currency balance sheet.
   - Off-balance-sheet hedging occurs through the purchase of forwards for institutions that choose to remain unhedged on the balance sheet.

7. Since domestic and foreign interest rates and stock returns do not usually move in perfect correlation, opportunities for potential gains from asset-liability portfolio diversification can offset currency risk.

Concept Checkers

1. Ion National Bank issues a 6-month, USD1 million CD at 4.0% and funds a loan in Argentine pesos (ARS) at 6.50%. The spot rate for the ARS was ARS2.27 per USD at the time of the transaction. In 6 months, the ARS will have depreciated to ARS2.30 per USD. What is the realized nominal annual spread on the loan?
   A. –1.07%.
   B. –0.19%.
   C. 0.11%.
   D. 0.13%.

2. With respect to Japanese yen (JPY), a U.S. firm has exchange-rate risk:
   A. that depends only on its net asset-liability position.
   B. if its JPY-denominated bonds have greater value than its JPY-denominated loans.
   C. only if its net JPY position is positive.
   D. whenever its total JPY assets are not equal to its total JPY liabilities.

Use the following data to answer Questions 3 through 5.

Century Bank issues USD20 million in U.S. CDs to fund its loan portfolio. The following characteristics pertain to the asset-liability position of the bank:

- A promised 1-year rate on the CDs of 7%.
- It invests 50% of its USD20 million in 1-year U.K. loans at 12% (loans made in GBP).
- The bank invests the other 50% in U.S. loans at 8% for one year.
- At the beginning of the year, the bank sells USD10 million for GBP in the spot currency markets at an exchange rate of USD1.42/GBP.
- The 1-year forward exchange rate is USD1.40/GBP.

3. If the spot foreign exchange rate does not change over the year, the USD proceeds from the U.K. investment will be:
   A. USD7,040,000.
   B. USD7,890,000.
   C. USD11,200,000.
   D. USD12,000,000.

4. If the exchange rate falls to USD1.38/GBP, what is the weighted return on the bank's asset portfolio?
   A. 1.41%.
   B. 2.82%.
   C. 5.41%.
   D. 8.42%.

5. If the bank hedges its GBP loan in the forward market, what is the return on the bank's loan portfolio?
   A. 8.37%.
   B. 9.21%.
   C. 9.79%.
   D. 10.11%.

Concept Checker Answers

1. B USD1 M × ARS2.27 × 1.0325 = ARS2,343,775 / 2.30 = USD1,019,033 – (USD1 M × 1.02) = –$967.40; –967.40/1 M = –0.0009674 × 2 = –0.19%

2. D A firm's exchange-rate risk depends on its net asset-liability exposure and on the volatility of the exchange rate with the JPY. Bonds and loans are only part of the whole JPY-denominated portfolio; forward contracts and currency holdings must be included to calculate the net asset-liability exposure. Either a positive or negative imbalance between JPY-denominated assets and liabilities will expose the firm to exchange rate risk.

3. C USD1.42/GBP = USD10,000,000 / 1.42 = GBP7,042,254 (1.12) = GBP7,887,324 × 1.42 = USD11,200,000

4. D USD10,000,000 × 1 / 1.42 = GBP7,042,254

   GBP7,042,254 × 1.12 × USD1.38/GBP = USD10,884,507

   (USD10,884,507 – USD10,000,000) / 10,000,000 = 0.08845 = 8.845%

   (0.5)(0.08) + (0.5)(0.08845) = 8.42%

5. B USD 10,000,000 × 1 / 1.42 = GBP7,042,254 × 1.12 × USD 1.40/GBP = USD 11,042,254; (11,042,254 – 10,000,000) / 10,000,000 = 10.42%; (0.5)(0.08) + (0.5)(0.1042) = 9.21%

The following is a review of the Financial Markets and Products principles designed to address the AIM statements set forth by GARP®. This topic is also covered in:

Corporate Bonds

Topic 35

Exam Focus

The term "bond" refers to a variety of assets which offer a wide range of interest rate payments from fixed cash payments, to accruals without cash, to payments in the form of additional securities. In this topic, we will provide an overview of major fixed-income instruments and their payment structures. We will also address the impact of credit risk and event risk on bond ratings and features. For the exam, be familiar with the types of bonds discussed and the methods for retiring bonds. Also, know the terminology associated with high-yield issues.

Bond Indenture and Role of Corporate Trustee

AIM 35.1: Describe a bond indenture and explain the role of the corporate trustee.

The **bond indenture** is a document that sets forth the obligation of the issuer and the rights of the investors in the bonds (i.e., the bondholders). It is usually a detailed document filled with legal language. One of the roles of the **corporate trustee** is to interpret this language and represent the interests of the bondholders. Banks or trust companies most often serve as corporate trustees, and the position requires that they act in a fiduciary capacity on behalf of the bondholders. The trustee would authenticate the issue, which includes keeping track of the amount of bonds issued and making sure the number does not exceed the limit specified in the indenture. The trustee would monitor the corporation's activities to make sure the issuer abides by the indenture's covenants (e.g., maintaining key ratios below a given number).

All corporate bond offerings over $5 million and sold in interstate commerce must have a corporate trustee as set forth in the **Trust Indenture** Act. The corporate trustees must be competent and financially responsible and should also not have any conflicts of interest, (e.g., being a creditor of the issuer). The indenture would specify how the trustee would make reports to bondholders and what to do if the issuer fails to pay interest or principal. As mentioned earlier, the basic goal of the trustee is to protect the rights of bondholders.

Maturity Date

AIM 35.2: Explain a bond's maturity date and how it impacts bond retirements.

The maturity date of a bond is when the bond issuer's obligations are fulfilled. At maturity, the issuer pays the principal and any accrued interest or premium. The contract, as set forth by the indenture, may terminate prior to the maturity date if, for example, the corporation chooses to retire the bonds early. The longer the maturity of the bond, the more time a company has to retire the bond issue.

Interest Payment Classifications

AIM 35.3: Describe the main types of interest payment classifications.

AIM 35.4: Describe zero-coupon bonds, the relationship between original-issue-discount and reinvestment risk, and the treatment of zeroes in bankruptcy.

The main types of bond interest payment classifications are: straight-coupon bonds, zero-coupon bonds, and floating-rate bonds. The interest rate on a bond is often called the **coupon**. However, bonds today technically no longer have coupons attached directly to them. Now, bonds are registered and represented by a certificate, or they are kept in book-entry form where one master or global certificate is issued and held by a central securities depository that issues receipts. This method is considered a safer way to make payments.

Straight-coupon bonds, also called fixed-rate bonds, have a fixed interest rate set for the entire life of the issue. In the United States, fixed-rate bonds typically pay interest every six months. In Europe and some other countries, bonds make annual interest rate payments. A bond issued in the United States with an 8% interest rate and a $1,000 par value on March 1, 2009 will pay $40 of interest each September 1 and each March 1 until its maturity date or until the bond is retired, at which time the issuer would pay both the final interest payment and the $1,000 principal back to the bondholder.

In addition to just paying a fixed dollar interest, bonds in the United States have been issued that pay in foreign currency. Two other variations are a participating bond and an income bond. **Participating bonds** pay at least the specified interest rate but may pay more if the company's profits increase. **Income bonds** pay at most the specified interest, but they may pay less if the company's income is not sufficient. In both cases, the conditions for paying more or less than the specified coupon would be set forth in the indenture.

Floating-rate bonds are also known as variable rate bonds. The interest paid is generally linked to some widely used reference rate such as LIBOR or the Federal Funds rate.

Zero-coupon bonds pay the face value or principal at maturity. There is not a cash interest payment; instead, the bondholder earns a return by purchasing the bond at a discount to face value and receiving the full face value at maturity. Variations of the zero-coupon bond include the **deferred-interest bond** (DIB) and the **payment-in-kind bond** (PIK). The DIB will not pay cash interest for some number of years early in the life of the bond. That period is the deferred-interest period. During this period, cash interest accrues and is then paid semiannually until maturity or when redeemed. PIK bonds pay interest with additional bonds for the initial period, and then cash interest after that period ends.

Most zero-coupons issued today share a host of other features such as being convertible, callable, and putable. A zero-coupon bond's interest rate is determined by the original-issue discount (OID):

original-issue discount (OID) = face value – offering price

The value of the bond grows each year and thus pays implicit interest, which is a function of the OID and the term-to-maturity. In other words, the rate of return depends on the amount of the discount and the period over which it grows.

One advantage of zero-coupon bonds is zero **reinvestment risk**. The bondholder does not have to make an effort to reinvest cash interest payments or worry about the available rates at which to reinvest them. A disadvantage is that the bondholder must pay taxes each year on the accrued interest even though no cash is received from the bond issuer.

If the issuer goes into bankruptcy prior to the maturity of a zero-coupon bond, the bondholders are only entitled to the accrued interest up to that date and not the full face value of the bond. In other words, the zero-coupon bond creditor can only claim the original offering price plus accrued and unpaid interest up to the date of the bankruptcy filing. The bond issuer faces a huge liability with a zero-coupon bond because of the large balloon payment at maturity.

Bond Types

AIM 35.5: Describe the various security types relevant for corporate bonds, including:
- **Mortgage bonds**
- **Collateral trust bonds**
- **Equipment trust certificates**
- **Debenture bonds (including subordinated and convertible debentures)**
- **Guaranteed bonds**

Corporate bonds can have a security, such as real property, underlying the issue. Those who own mortgage bonds have a first-mortgage lien on the properties of the issuer. This security allows the issuer to pay a lower rate of return than it would have to pay on unsecured bonds, which are known as debentures. The lien gives the bondholders the right to sell the mortgaged property to satisfy unpaid obligations to bondholders. In practice, this right is usually used for bargaining purposes only, and the bankruptcy takes the form of reorganization as opposed to liquidation.

Mortgage bonds can be issued in a series in a blanket arrangement. In this case, one group of bonds is issued under the mortgage, and then others are issued later. When earlier issues mature, additional bonds are then issued in their place.

Collateral trust bonds are backed by stocks, notes, bonds, or other similar obligations that the company owns. The underlying assets are called the collateral or personal property. The issuers are holding companies, and the collateral consists of claims on their subsidiaries.

The trustee holds the collateral for the benefit of the bondholders; however, the issuer retains voting rights for stock used as collateral, so they retain control over their subsidiaries. The indenture may have provisions covering what to do if the value of the collateral falls below the value of the loan. If the collateral falls in value, the issuer may have to contribute additional securities to back the bonds. The issuer may be able to withdraw collateral if the value rises in order to exceed the loan value. Like mortgage bonds, collateral trust bonds may be issued in series.

Equipment trust certificates (ETCs) are a variation of a mortgage bond where a particular piece of equipment underlies the bond. The usual arrangement is that the borrower does not actually purchase the equipment. Instead, the trustee purchases the equipment and leases it to the user of the equipment (the effective borrower), who pays rent on the equipment, and that rent is passed through to the holders of the ETCs. The payments to the creditors are called **dividends**. The trustee pays for the equipment with the money raised from the issuance of the ETCs, usually about 80% of the value of the equipment, and what is effectively a down payment from the user of the equipment. This provides more security to the creditors than that of a mortgage bond. It is especially attractive if the equipment is standardized, as in the case of railroad cars, which provides for easy sale or lease of the equipment in the case the user of the equipment defaults. ETCs are generally considered the most secure type of bond since the underlying assets are actually owned by the trustee and rented to the borrower.

As noted earlier, **debentures** are unsecured bonds (i.e., they do not have any assets underlying the issue). Most corporate bonds are debentures and usually pay a higher interest rate for that reason. However, if the company is highly rated and has not issued any secured bonds, then debentures are almost the equivalent of mortgage bonds in that they have a claim on all the assets of the issuer along with the general creditors. If the issuer has issued secured debt along with debentures, the debenture holders have a claim on the assets that are not backing the secured debt. Typically the issuer is restricted to one issue of debentures if there is already secured debt. If there is no secured debt, and the company issues debentures, there is often a negative-pledge clause that says that the debentures will be secured equally with any secured bonds that may be issued in the future.

Subordinated debenture bonds have a claim that is at the bottom of the list of creditors if the issuer goes into default. They are bonds that are unsecured and have another unsecured bond with a higher claim above them. This means that the issuer has to offer a higher interest rate on the subordinated debentures.

Issuers may choose to issue **convertible debentures**, which give the bondholder the right to convert the bond into common stock. This feature will lower the interest rate paid. The cost to the issuer, however, is the possibility of increased dilution of the stock. A variation of convertible debentures is **exchangeable debentures** that are convertible into the common stock of a corporation other than that of the issuer.

Bonds issued by one company may also be guaranteed by other companies. These bonds are known as **guaranteed bonds**. A guarantee does not ensure that the issue will be free of default risk since the risk will depend on the ability of the guarantor(s) to satisfy all obligations.

Methods for Retiring Bonds

AIM 35.6: Describe the mechanisms by which corporate bonds can be retired before maturity, including:
- **Call provisions**
- **Sinking-fund provisions**
- **Maintenance and replacement funds**
- **Tender offers**

There are a variety of methods for retiring debt, and some are included in the bond's indenture while others are not included. The indenture would include the call and refunding provisions, sinking funds, maintenance and replacement funds, and redemption through sale of assets. The indenture would not include fixed-spread tender offers.

Call and refunding provisions are essentially call options on the bonds that the issuer owns and give the issuer the right to purchase at a fixed price either in whole or in part prior to maturity. These provisions allow a firm to call back debt that has a high coupon and reissue debt with a lower coupon. Other reasons for exercising these options are to eliminate restrictive covenants, alter capital structure, increase shareholder value, or improve financial/managerial flexibility. A **call provision** can either be a fixed-price call or a make-whole call.

- **Fixed-price call.** The firm can call back the bonds at specific prices that can vary over the life of the bonds as specified in the indenture. They generally start out high and decline toward par. Also, for most bonds, the bonds are not callable during the first few years of the issue's life.
- **Make-whole call.** In this case, market rates determine the call price, which is the present value of the bond's remaining cash flows subject to a floor price equal to par value. A discount rate based on the yield of comparable-maturity Treasury securities (usually the rate plus a premium) determines the present value and the bond's price. The redemption price is the greater of that present value or the par value plus accrued interest.

A **sinking fund provision** generally means the issuing firm retires a specified portion of the debt each year as outlined in the indenture. The bonds can either be retired by use of a lottery where the owners of the selected bonds must redeem them, or the bonds are purchased in the open market. The purchase of some sufficient amount of equipment in excess of the value of the amount of the bonds to be retired is another action that may satisfy a sinking fund provision.

The lottery approach to satisfying the sinking fund is very similar to a call provision in that the bondholders must sell back their bonds at a specified price. Unlike the call provision, there may be advantages to the bondholders. First, the retirement of bonds improves the financial health of the firm. Second, the redemption price may exceed the market price. However, the indenture may give the issuer some flexibility as to how much of the bonds to call back each period, which would give the firm some latitude to call back more bonds when the market conditions are favorable to do so. One example is an accelerated sinking-fund provision, which allows the firm to call back more bonds in early years, which the firm would do if interest rates fall in those early years.

A **maintenance and replacement fund** (M&R) has the same goal as a sinking fund provision, which is to maintain the credibility of the property backing the bonds. The provisions differ in that the M&R provision is more complex since it requires valuation

formulas for the underlying assets. The main point is that the provision specifies that the fund must keep up the value of the underlying assets much like a home mortgage specifies the home buyer must keep up the value of the home. One way to satisfy the provision is to acquire sufficient cash to maintain the health of the firm. That cash can then be used to retire debt.

Tender offers are usually a means for retiring debt for most firms. The firm openly indicates an interest in buying back a certain dollar amount of bonds or, more often, all of the bonds at a set price. The goal is to eliminate restrictive covenants or to use excess cash. If the first tender offer price does not get sufficient interest, the firm can increase its offer price. Firms can also announce that they will buy back bonds based on the price as determined by a certain market interest rate (e.g., the yield to maturity on a comparable-maturity Treasury plus a spread). This lowers interest rate risk for both the bondholders and the bond issuer.

As a final note, the issuing firm may be able to call back bonds if it is necessary to sell assets associated with the bond issue. For example, if the government requires a firm to sell property, but that property is being used as collateral for the bonds, the firm would sell the property and call back the bonds.

Credit Risk

AIM 35.7: Describe and differentiate between credit default risk and credit spread risk.

Credit risk includes credit default risk and credit spread risk. **Credit default risk** is the uncertainty concerning the issuer making timely payments of interest and principal as prescribed by the bond's indenture. The most widely-used indicators of this risk are bond ratings that major rating agencies assign when those agencies perform credit analysis of a firm. Fitch Ratings, Moody's, and Standard & Poor's are the main rating agencies in the United States. The agencies assign a symbol associated with the rating (e.g., AAA or Aaa for the corporate debt with the least credit default risk). The rating can be interpreted as a probability of default within some time period.

Professor's Note: We will discuss credit ratings in more detail in Book 4.

Credit spread risk focuses on the difference between a corporate bond's yield and the yield on a comparable-maturity benchmark Treasury security. This difference is known as the **credit spread**. It should be noted that other factors such as embedded options and liquidity factors can affect this spread; therefore, it is not only a function of credit risk.

The risk here is from possible changes in this spread from changes in investor risk aversion, which will change the value of the associated bond. Other factors affecting credit spreads are macroeconomic forces such as the level and slope of the Treasury yield curve, the business cycle, and issue-specific factors such as the corporation's financial position and the future prospects of the firm and its industry.

A method commonly used to evaluate credit spread risk is **spread duration**. The duration of the spread is the approximate percentage change in a bond's price for a 100 basis point change in the credit spread assuming that the Treasury rate is constant. If a bond has a spread duration of 4, for example, a 50 basis point change in the spread will change the value of the bond by 2%.

Event Risk

AIM 35.8: Describe event risk and what may cause it in corporate bonds.

Event risk addresses the adverse consequences from possible events such as mergers, recapitalizations, restructurings, acquisitions, leveraged buyouts, and share repurchases, which may escape being included in the indenture. Such events can drastically change the firm's capital structure and reduce the creditworthiness of the bonds and their value. In order to protect shareholders, a company may include in the indenture a **poison put**, which can require the company to repurchase the debt at or above par value in the event of a takeover not approved by the board of directors (i.e., a hostile takeover). The purpose of this feature is to protect bondholders, but its effectiveness toward this goal can be misleading in that the acquiring firm may offer a sufficiently high price for the stock so that the hostile takeover becomes friendly. As a result, the poison puts would not be exercised.

Investors can lobby for clauses in the indenture to activate a put option for a variety of reasons including a change in the bond's rating. Some of the debt rating services issue commentary on the indenture's protective features, which could include the possibility of the firm being able to circumvent the features through careful legal moves (e.g., turning a hostile takeover into a friendly takeover). It should be noted that event risk can change on a market level. During times of increased merger activity, for example, the event risk increases for most bonds.

High-Yield Bonds

AIM 35.9: Define high-yield bonds, describe types of high-yield bond issuers, and some of the payment features peculiar to high-yield bonds.

High-yield bonds (a.k.a. junk bonds) are those bonds rated below investment grade by ratings agencies. This includes a broad range of ratings below the cutoff, (e.g., BB to default). **Businessman's risk** refers to bonds with a rating at the bottom rung of the investment-grade category (Baa and BBB) or at the top end of the speculative-grade category (Ba and BB). Over long periods of time, high-yield bonds should offer higher average returns. However, over shorter periods, the returns will be volatile where large losses are possible.

There are many types of high-yield bonds. One type includes companies who issue bonds with a non-investment grade rating. Such issuers include young and growing companies that do not have strong financial statements but who have promising prospects. Firms may issue such bonds to raise venture capital, and their prospects are tied to a particular project or story, which gives them the name "story bonds."

Established firms that have had a deteriorating financial situation may need to raise debt capital as well, and they would issue bonds that reflect their situation. Also, an established firm who already has unsecured debt issued with an investment-grade credit rating may be able to issue subordinated debt, but that debt would be non-investment grade.

Fallen angels are another type of high-yield bond. They are bonds that were issued with an investment-grade rating, but then events led to the ratings agencies lowering the rating to below investment grade. If the issuers are in or near bankruptcy, they are often called "special situations," which could either pay off if the company recovers or lead to big losses.

Restructurings and **leveraged buyouts** may increase the credit risk of a company to the point where the bonds become non-investment grade. The new management may pay high dividends, deplete the acquired firm's cash, and lower the rating of the existing bonds. In this process, the firm may issue non-investment grade debt to pay off the bridge loans taken to finance the acquisition.

High-yield bonds can have several types of coupon structures. There are **reset bonds**, where designated investment banks periodically reset the coupon to reflect market rates and the creditworthiness of the issuer. There are also **deferred-coupon structures**, which include three types: (1) deferred-interest bonds, (2) step-up bonds, and (3) payment-in-kind bonds. Deferred-interest bonds sell at a deep discount and do not pay interest in the early years of the issue, say, for three to seven years. Step-up bonds pay a low coupon in the early years and then a higher coupon in later years. Payment-in-kind bonds allow the issuer to pay interest in the form of additional bonds over the initial period.

Default Rate

AIM 35.10: Define and differentiate between an issuer default rate and a dollar default rate.

A default occurs if there are any missed or delayed disbursements of interest and/or principal. It has been proven that lower credit ratings indicate a higher probability of default, but there are two ways to measure default: by the raw number of issuers that defaulted or the dollar amount of issues that defaulted. For each approach in measuring default rates, there are different formulas, which can lead to researchers reporting different default rates for the same data set.

The **issuer default rate** is the number of issuers that defaulted over a year divided by the total number of issuers at the beginning of the year. It is only a proportion of the number of issuers who do fulfill their obligations and does not include a measure of the dollar amount involved.

The **dollar default rate** is the par value of all bonds that defaulted in a given calendar year divided by the total par value of all bonds outstanding during the year. Over a multi-year period, often-used measures are ratios of cumulative dollar value of all defaulted bonds

divided by some weighted-average measure of all bonds issued. One such measure attempts to weight the bonds outstanding by the number of years they are in the market:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{(\text{cumulative dollar value of all issuance}) \times (\text{weighted average \# of years outstanding})}$$

Another measure simply takes a raw total as shown in the following equation:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{\text{cumulative dollar value of all issuance}}$$

Recovery Rate

AIM 35.11: Define recovery rates and describe the relationship between recovery rates and seniority.

The recovery rate is the amount received as a proportion of the total obligation after a bond defaults. Measuring this can be complicated because the value of the total obligation requires computing the present value of the remaining cash flows at the time of the default. Furthermore, some of the amount that the investor recovers may be in the form of securities (e.g., stock in the company). A study by Moody's estimated that the recovery rate for bonds has been about 38%. Bonds with higher seniority will obviously have higher recovery rates.

Key Concepts

1. A bond indenture sets forth the obligations of the issuer. The trustee interprets the legal language of the indenture and works to make sure the issuer fulfills obligations to bondholders.

2. The bond issuer's obligations are fulfilled on the maturity date or before. Bonds can be retired before that date.

3. The main types of interest payment classifications are straight-coupon bonds, zero-coupon bonds, and floating-rate bonds. Straight-coupon bonds pay a fixed cash coupon periodically. Floating-rate bonds pay a cash amount that varies with market rates. Zero-coupon bonds increase in value over the life of the issue.

4. There are many variations of the main types of bond structures. For example, deferred-interest bonds are a mix of zero-coupon and coupon bonds in that they do not pay cash interest in early years and pay a cash coupon in later years. Some bonds have principal in one currency and pay coupons in another currency.

5. Zero-coupon bonds have low reinvestment risk. The interest is based on the time-to-maturity at issuance and the original-issue discount, which is the difference between the face value and the offering price. In the case of bankruptcy, the bondholder has a claim only equal to the issue price plus accrued interest to that date, and not the full face value.

6. The holder of a mortgage bond has the first lien on real property owned by the issuer.

7. Collateral trust bonds are backed by stocks and bonds that represent claims against the subsidiaries of the issuer. The collateral is also called personal property.

8. Equipment trust certificates are a form of mortgage bond where the trustee actually owns the property and rents it to the bond issuer. The property is often in the form of standardized equipment (e.g., rail cars) that is easily sold.

9. Debentures are unsecured debt. Owners of debentures have a claim on the company's assets not backing outstanding secured debt.

10. Call provisions allow the firm to retire debt early at a given price. Sinking-fund provisions require the firm to buy back portions of debt each year. Call provisions are generally considered detrimental to bondholders, but sinking-fund provisions may be beneficial to bondholders.

11. A maintenance and replacement fund helps maintain the financial health of the firm. Cash in the fund can be used to retire debt.

12. Bond issuers can retire debt through a tender offer. The offer price may either be a fixed price or a price that varies with a market rate such as that on comparable Treasury securities.

13. Credit default risk is the possibility that the issuer does not make the payments specified in the indenture. Credit spread risk is the price risk from changes in the spread of a bond's interest rate over the corresponding Treasury rate.

14. Event risk is the possibility that a merger, restructuring, acquisition, et cetera, increases the risk of the bond by changing the ability of the firm to pay off the bonds. The indenture can try to address some of these events, but some can be omitted and lawyers can find loopholes around those included.
15. High-yield bonds can either be issued by growing, risky firms or established firms with senior debt outstanding. High-yield bonds may also be fallen angels (i.e., one-time investment grade bonds).
16. High-yield bonds may have coupon structures which allow the firm to conserve cash in early years, such as: (1) deferred-interest bonds, (2) step-up bonds, and (3) payment-in-kind bonds.
17. The issuer default rate is a proportion based on the number of issues that default as a proportion of all issues. The dollar default rate estimates the dollar amount of defaulted bonds compared to the dollar amount of the corresponding population of bonds outstanding.
18. In the event of default, the recovery rate refers to the amount a bondholder receives as a proportion of the amount owed. Bonds with higher seniority usually have higher recovery rates.

Concept Checkers

1. Which of the following responsibilities is least likely to be part of the role of a corporate trustee in a bond issue?
   A. Interpret the language of the indenture.
   B. Determine the interest rate on a reset bond.
   C. Keep track of the amount of bonds issued by the corporation.
   D. Monitor the corporation's activities to make sure the corporation abides by the indenture's covenants.

2. In bankruptcy, the holder of a zero-coupon bond obligation of the bankrupt corporation would have a claim equal to:
   A. the face value of the bond.
   B. the issuing price of the bond only.
   C. the issuing price plus accrued interest.
   D. nothing, since zeros are always unsecured.

3. All other things being equal, which of the following types of bond instruments would have the lowest interest rate?
   A. Equipment trust certificates.
   B. Mortgage bonds.
   C. Junior debentures.
   D. Senior debentures.

4. Which of the following methods for retiring bonds before maturity is generally considered the most detrimental for the bondholders?
   A. Tender offers.
   B. Call provision.
   C. Sinking fund provision.
   D. Maintenance and replacement funds.

5. With respect to default risk and credit spread risk, the ratings of bond-rating agencies such as Moody's provide information concerning:
   A. default risk only.
   B. credit spread risk only.
   C. both default risk and credit spread risk.
   D. neither default risk nor credit spread risk.

Concept Checker Answers

1. **B** Investment banks other than the trustee set the rate on a reset bond.

2. **C** The claim equals the value at that point in time as implied by the issuing price, the original-issue discount, and accrued interest.

3. **A** ETCs, or equipment trust certificates, are generally the most secure because the underlying assets are actually owned by the trustee and rented to the borrower. Also, the assets are usually standardized for easy resale.

4. **B** The call provision gives the issuer the right to purchase the bonds at a given price, which the issuer would not do unless that price was below the market price. Sinking fund provisions can benefit bondholders because the issuer is obligated to purchase bonds, which improves the creditworthiness of the issue, and the issuer may have to do so at a price higher than the market price. There are no features in M&R funds or tender offers that would be detrimental to bondholders.

5. **A** Bond rating agencies issue ratings based on the default risk of the issue. Credit spread risk is determined by spread duration.

Challenge Problems

1. An investor enters a short position in a gold futures contract at \$318.60. Each futures contract controls 100 troy ounces. The initial margin is \$5,000 and the maintenance margin is \$4,000. At the end of the first day the futures price rises to \$329.22. Which of the following is the amount of the variation margin at the end of the first day?
   A. \$0.
   B. \$62.
   C. \$1,000.
   D. \$1,062.

2. A large-cap U.S. equity portfolio manager is concerned about near-term market conditions and wishes to reduce the systematic risk of her portfolio from 1.2 to 0.90. Her portfolio value is \$56 million, and the S&P 500 futures index is currently trading at 1,050 and has a multiplier of 250. How can the portfolio manager's objective be achieved?
   A. Sell 47 contracts.
   B. Buy 47 contracts.
   C. Sell 64 contracts.
   D. Buy 64 contracts.

3. Suppose you observe a 1-year (zero-coupon) Treasury security trading at a yield to maturity of 5% (price of 95.2381% of par). You also observe a 2-year T-note with a 6% coupon trading at a yield to maturity of 5.5% (price of 100.9232). And, finally, you observe a 3-year T-note with a 7% coupon trading at a yield to maturity of 6.0% (price of 102.6730). Assume annual coupon payments. Use the bootstrapping method to determine the 2-year and 3-year spot rates.

	2-year spot rate	3-year spot rate
A.	5.51%	5.92%
B.	5.46%	5.92%
C.	5.51%	6.05%
D.	5.46%	6.05%

4. Former Treasury Secretary Robert Rubin decided to stop issuing 30-year Treasury bonds in 2001 and to replace them by borrowing more with shorter-maturity Treasury bills and notes (although the U.S. Treasury has since resumed issuing 30-year bonds). Which of the following statements concerning this decision is most accurate?
   A. If the pure expectations hypothesis of the term structure is correct, this decision will reduce the government's borrowing cost.
   B. If the liquidity theory of the term structure is correct, this decision will reduce the government's borrowing cost.
   C. If the liquidity theory of the term structure is correct, this decision will not change the government's borrowing cost.
   D. If the pure expectations hypothesis of the term structure is correct, this decision will increase the government's borrowing cost.

5. A portfolio manager owns Macrogrow, Inc., which is currently trading at $35 per share. She plans to sell the stock in 120 days but is concerned about a possible price decline. She decides to take a short position in a 120-day forward contract on the stock. The stock will pay a $0.50 per share dividend in 35 days and $0.50 again in 125 days. The risk-free rate is 4%. The value of the trader's position in the forward contract in 45 days, assuming in 45 days the stock price is $27.50 and the risk-free rate has not changed, is closest to:
   A. $7.17.
   B. $7.50.
   C. $7.92.
   D. $7.00.

6. A 6-month futures contract on an equity index is currently priced at 1,276. The underlying index stocks are valued at 1,250 and pay dividends at a continuously compounded rate of 1.70%. The current continuously compounded risk-free rate is 5%. The potential arbitrage is closest to:
   A. 5.20.
   B. 8.32.
   C. 16.58.
   D. 26.00.

7. All things being equal, which of the following statements would increase the value of a European put option on a nondividend paying stock?

   I. A decrease in the risk-free rate over the life of the option.
   II. An increase in the time to expiration of the option.

   A. I only.
   B. II only.
   C. Both I and II.
   D. Neither I nor II.

Use the following information to answer Questions 8 and 9.

Stock ABC trades for $60 and has 1-year call and put options written on it with an exercise price of $60. The annual standard deviation estimate is 10%, and the continuously compounded risk-free rate is 5%. The value of the call is $4.09.

Chefron, Inc. common stock trades for $60 and has a 1-year call option written on it with an exercise price of $60. The annual standard deviation estimate is 10%, the continuous dividend yield is 1.4%, and the continuously compounded risk-free rate is 5%.

8. The value of the put on ABC stock is closest to:
   A. $1.16.
   B. $3.28.
   C. $4.09.
   D. $1.00.

9. The value of the call on Chefron stock is closest to:
   A. \$3.51.
   B. \$4.16.
   C. \$5.61.
   D. \$6.53.

10. One of your clients, Christopher Stachowski, realizes that the market prices of options must take into account the beliefs of the market participants. He thinks he will be able to make significant profits because he believes that there will be a large movement in the direction of stock prices but is unsure which direction. Such a belief is completely different from the other market participants. As a result, Christopher would like you to implement an options trading strategy to generate him those profits. Which of the following combination option strategies is likely to benefit the least amount from a large positive or negative movement in the price of the underlying?
    A. Strip.
    B. Strap.
    C. Collar.
    D. Long strangle.

11. You are a portfolio manager and have just been asked by one of your junior colleagues to discuss the replication of specific payoffs using synthetic instruments. She begins by asking you how to replicate the payoff diagram for a short put position. Such a position could be replicated using which of the following strategies?
    A. Covered call.
    B. Bear put spread.
    C. Protective put.
    D. Bull call spread.

12. Consider a bearish option strategy of buying one \$50 put for \$7, selling two \$42 puts for \$4 each, and buying one \$37 put for \$2. All the options have the same maturity. Calculate the final profit per share of the strategy if the underlying is trading at \$33 at expiration.
    A. \$1 per share.
    B. \$2 per share.
    C. \$3 per share.
    D. \$4 per share.

13. Suppose the spot rate is 0.7102 USD/CHF. Swiss and U.S. interest rates are 7.6% and 5.2%, respectively. If the 1-year forward rate is 0.7200 USD/CHF, an investor could:
    A. not earn arbitrage profits.
    B. earn arbitrage profits by investing in USD.
    C. earn arbitrage profits by investing in CHF.
    D. only earn arbitrage profits by investing in a third currency.

14. Consider a U.K.-based company that exports goods to the EU. The U.K. company expects to receive payment on a shipment of goods in 60 days. Because the payment will be in euros, the U.K. company wants to hedge against a decline in the value of the euro against the pound over the next 60 days. The U.K. risk-free rate is 3% and the EU risk-free rate is 4%. No change is expected in these rates over the next 60 days. The current spot rate is 0.9230 £ per €. To hedge the currency risk, the U.K. company should take a short position in a Euro contract at a forward price of:
A. 0.9205.
B. 0.9215.
C. 0.9244.
D. 0.9141.

Challenge Problem Answers

1. D The short position loses when the price rises.

   (\$329.22 – \$318.60) × 100 = 1,062 loss

   Margin account will change as follows: \$5,000 – \$1,062 = \$3,938

   Variation margin of \$1,062 is required because the balance has fallen below the maintenance margin level. This variation margin payment is required in order to restore the account back to the initial level.

   (See Topic 24)

2. C The portfolio manager wants to reduce exposure to systematic risk so she will want to sell S&P index futures. This will reduce the current beta to her target beta of 0.90.

   number of contracts = (target beta – current beta) × (portfolio value / futures value)

   number of contracts = (0.9 – 1.2) × [\$56 million / (1,050 × 250)]

   number of contracts = –64 (i.e., sell 64 contracts)

   (See Topic 25)

3. C Here are the cash flows associated with the three bonds:

	0	1	2	3
1-year	–\$95.2381	+\$100		
2-year	–\$100.9232	+\$6	+\$106	
3-year	–\$102.6730	+\$7	+\$7	+\$107

   To find Z_2, the 2-year spot rate:

$$\$100.9232 = \frac{\$6}{1.05^1} + \frac{\$106}{(1+Z_2)^2} \Rightarrow Z_2 = 5.51\%$$

   To find Z_3, the 3-year spot rate:

$$\$102.6730 = \frac{\$7}{1.05^1} + \frac{\$7}{1.0551^2} + \frac{\$107}{(1+Z_3)^3} \Rightarrow Z_3 = 6.05\%$$

   (See Topic 26)

4. **B** If the pure expectations hypothesis of the term structure is correct, altering the maturity of the government's borrowing will not affect the government's borrowing cost (i.e., borrowing once for 30 years is the same as borrowing 30 times for one year at a time). If the liquidity theory is correct, the government's borrowing cost will go down, as it no longer has to compensate lenders with the liquidity premium for borrowing long term.

   (See Topic 26)

5. A The dividend in 125 days is irrelevant because it occurs after the forward contract matures.

$$PVD = \frac{\$0.50}{1.04^{35/365}} = \$0.4981$$

$$FP = (\$35 - \$0.4981) \times 1.04^{120/365} = \$34.95$$

$$V_{45}(\text{short position}) = -\left(\$27.50 - \frac{\$34.95}{1.04^{75/365}}\right) = \$7.17$$

   (See Topic 27)

6. A $F = S \times e^{(\text{risk-free rate} - \text{dividend yield}) \times t}$

   $F = 1{,}250 \times e^{(0.05 - 0.017) \times 0.5}$

   $F = 1{,}270.80$

   The actual futures price is 1,276, so selling the futures and buying the underlying index nets a profit of 1,276 – 1,270.80 = 5.20.

   (See Topic 27)

7. A As the risk-free rate decreases (increases), the value of a European put option increases (decreases). Therefore, there is an inverse relationship between the value of a European put option and the risk-free rate. The opposite relationship exists for European call options.

   For a nondividend paying stock, the value of a European put option would not be affected by the time to expiration since the option may only be exercised on the expiration date. Had it been a dividend paying stock, and there was a dividend paid during the life of the option, the value of the underlying stock would decrease and, therefore, the value of the put would increase. Also, had it been an American put option, increasing the time to expiration would increase the value of the option since the additional time increases the likelihood of the option being in the money.

   (See Topic 30)

8. A According to put/call parity, the put's value is:

$$p_0 = c_0 - S_0 + \left(X \times e^{-R_c^f \times T}\right) = \$4.09 - \$60.00 + \left[\$60.00 \times e^{-(0.05 \times 1.0)}\right] = \$1.16$$

   (See Topic 30)

9. A ABC and Chefron stock are identical in all respects except Chefron pays a dividend. Therefore, the call option on Chefron stock must be worth less than the call on ABC (i.e., less than \$4.09). \$3.51 is the only possible answer.

   (See Topic 30)

10. **C** A collar is the combination of a protective put and a covered call. Ignoring transaction costs, at levels below the put strike price or above the call strike price, the profit from a collar levels off. Between the put strike price and the call strike price, the profit level is gradually rising.

(See Topic 31)

11. **A** Ignoring transaction costs, a short put results in a payoff diagram where there are losses to the writer when the price of the underlying is below the strike price and no losses/gains when the price of the underlying is above the strike price.

Similarly, a covered call (long underlying, short call) results in the same payoff diagram as above. There would obviously be losses on the long position at prices below the strike price. And there would be no losses/gains at prices above the strike price because the option holder will likely exercise the option and the option writer will have to liquidate the long position of the underlying at the strike price.

(See Topic 31)

12. **B** Consider each option separately:

\$50 long put:	\$50 – \$33 = +\$17
\$42 short put:	\$42 – \$33 = –\$9 × 2 = –\$18
\$37 long put:	\$37 – \$33 = +\$4
Net cost of options:	(–7 + 8 – 2) = –\$1
Overall profit per share:	\$2 per share

(See Topic 31)

13. **C** Note that while the USD has the lower interest rate, it is also trading at a forward discount relative to the CHF. Since the USD will earn less interest *and* depreciate in value, we definitely want to invest in CHF (not in USD), and no calculation is necessary.

As an illustration of covered interest arbitrage, we have:

$$(1+R_A) < \frac{(1+R_B)(\text{forward rate})}{\text{spot rate}}$$

$$1.052 < \frac{(1.076)(0.72)}{0.7102} = 1.0908$$

Today:
(1) Borrow USD1 at 5.2% and purchase CHF at \$0.7102 to get \$1 / 0.7102 = 1.408 CHF at spot rate.
(2) Lend the purchased CHF at 7.6% and sell forward 1.5150 CHF at the forward rate of 0.7200 USD/CHF.

In one year:
(1) Use the proceeds of the savings account [(1.408)(1.076) = 1.5150 CHF] to purchase USD1.0908 at the forward rate (1.515 CHF × 0.72 USD/CHF).
(2) Pay off the loan of USD1 × 1.052 = USD1.052 and earn a riskless profit = USD1.0908 – USD1.052 = USD0.0388.

(See Topic 34)

14. **B** The U.K. company will be receiving euros in 60 days, so it should short the 60-day forward on the euro as a hedge. The no-arbitrage forward price is:

$$F_T = £0.923 \times \frac{1.03^{60/365}}{1.04^{60/365}} = 0.9215$$

(See Topic 34)

GARP FRM Practice Exam Questions

Financial Markets and Products

Professor's Note: The following questions are from the 2008–2011 GARP FRM Practice Exams.

1. Alan bought a futures contract on a commodity on the New York Commodity Exchange on June 1. The futures price was USD 500 per unit and the contract size was 100 units per contract. Alan set up a margin account with initial margin of USD 2,000 per contract and maintenance margin of USD 1,000 per contract. The futures price of the commodity varied as shown below. What was the balance in Alan's margin account at the end of day on June 5?

Day	*Futures Price (USD)*
June 1	497.30
June 2	492.70
June 3	484.20
June 4	471.70
June 5	468.80

A. –USD 1,120
B. USD 0
C. USD 880
D. USD 1,710

2. In late June, Simon purchased two September silver futures contracts. Each contract size is 5,000 ounces of silver and the futures price on the date of purchase was USD 18.62 per ounce. The broker requires an initial margin of USD 6,000 and a maintenance margin of USD 4,500. You are given the following price history for the September silver futures:

Day	*Futures Price (USD)*	*Daily Gain (loss)*
June 29	18.62	0
June 30	18.69	700
July 1	18.03	–6,600
July 2	17.72	–3,100
July 6	18.00	2,800
July 7	17.70	–3,000
July 8	17.60	–1,000

On which days did Simon receive a margin call?
A. July 1 only
B. July 1 and July 2 only
C. July 1, July 2 and July 7 only
D. July 1, July 2 and July 8 only

3. Which of the following statements about basis risk is incorrect?
   A. An airline company hedging exposure to a rise in jet fuel prices with heating oil futures contracts may face basis risk.
   B. Choices left to the seller about the physical settlement of the futures contract in terms of grade of the commodity, location, chemical attributes may result in basis risk.
   C. Basis risk exists when futures and spot prices change by the same amount over time and converge at maturity of the futures contract.
   D. Basis risk is zero when variances of both the futures and spot process are identical and the correlation coefficient between spot and futures prices is equal to one.

4. On Nov 1, Jimmy Walton, a fund manager of an USD 60 million US medium-to-large cap equity portfolio, considers locking up the profit from the recent rally. The S&P 500 index and its futures with the multiplier of 250 are trading at USD 900 and USD 910, respectively. Instead of selling off his holdings, he would rather hedge two-thirds of his market exposure over the remaining 2 months. Given that the correlation between Jimmy's portfolio and the S&P 500 index futures is 0.89 and the volatilities of the equity fund and the futures are 0.51 and 0.48 per year respectively, what position should he take to achieve his objective?
   A. Sell 250 futures contracts of S&P 500
   B. Sell 169 futures contracts of S&P 500
   C. Sell 167 futures contracts of S&P 500
   D. Sell 148 futures contracts of S&P 500

5. Which of the following is not a source of basis risk when using futures contracts for hedging?
   A. Differences between the asset whose price is being hedged and the asset underlying the futures contract.
   B. Uncertainty about the exact date when the asset being hedged will be bought or sold.
   C. The inability of managers to forecast the price of the underlying.
   D. The need to close the futures contract before its delivery date.

6. On Nov 1, Dane Hudson, a fund manager of an USD 50 million US large cap equity portfolio, considers locking up the profit from the recent rally. The S&P 500 index and its futures with the multiplier of 250 are trading at USD 1,000 and USD 1,100, respectively. Instead of selling off h1s holdings he would rather hedge his market exposure over the remaining 2 months. Given that the correlation between Dane's portfolio and the S&P 500 index futures is 0.92 and the volatilities of the equity fund and the futures are 0.55 and 0.45 per year respectively, what position should he take to achieve his objective?
   A. Sell 40 futures contracts of S&P 500
   B. Sell 135 futures contracts of S&P 500
   C. Sell 205 futures contracts of S&P 500
   D. Sell 355 futures contracts of S&P 500

7. A stock index is valued at USD 800 and pays a continuous dividend at the rate of 3% per year. The 6-month futures contract on that index is trading at USD 758. The continuously compounded risk free rate is 2.5% per year. There are no transaction costs or taxes. Is the futures contract priced so that there is an arbitrage opportunity? If yes, which of the following numbers comes closest to the arbitrage profit you could realize by taking a position in one futures contract?
   A. 38
   B. 40
   C. 42
   D. There is no arbitrage opportunity.

8. The yield curve is upward sloping. You have a short T-Bond interest rate futures position. The following bonds are eligible for delivery:

Bonds	*Spot-Price (USD)*	*Conversion Factor*	*Coupon Rate*
A	102.44	0.98	4%
B	106.59	1.03	5%
C	98.38	0.95	3%

The futures price is 103 17/32 and the maturity date of the contract is September 1. The bonds pay their coupon amount semi-annually on June 30 and December 31. With these data, the cheapest-to-deliver bond is:
   A. Bond A
   B. Bond B
   C. Bond C
   D. Insufficient information to determine.

9. The yield curve is upward sloping. You have a short T-Bond interest rate futures position. The following bonds are eligible for delivery:

Bonds	*Spot-Price (USD)*	*Conversion Factor*	*Coupon Rate*
A	102.40	0.8	4%
B	100.40	1.5	5%
C	99.60	1.1	3%

The futures price is USD 104 and the maturity date of the contract is September 1. The bonds pay their coupon amount semi-annually on June 30 and December 31. With these data, which bond is cheapest-to-deliver?
   A. Bond A
   B. Bond B
   C. Bond C
   D. Insufficient information to determine.

10. A bank had entered into a 3-year interest rate swap for a notional amount of USD 300 million, paying a fixed rate of 7.5% per year and receiving LIBOR annually. Just after the payment was made at the end of the first year, the continuously compounded 1-year and 2-year annualized LIBOR rates were 7% per year and 8% per year, respectively. The value of the swap at that time was closest to which of the following choices?
    A. USD –14 million
    B. USD –4 million
    C. USD 4 million
    D. USD 14 million

11. On the OTC market there are two options available on Microsoft stock: a European put with premium of USD 2.25 and an American call option with premium of USD 0.46. Both options have a strike price of USD 24 and an expiration date 3 months from now. Microsoft's stock price is currently at USD 22 and no dividend is due during the next 6 months. Assuming that there is no arbitrage opportunity, which of the following choices is closest to the level of the risk-free rate:
    A. 0.25%
    B. 1.76%
    C. 3.52%
    D. Insufficient information to determine.

12. A stock is trading at USD 100. A box spread with 1 year to expiration and strikes at USD 120 and USD 150 is trading at USD 20. The price of a 1-year European call option with strike USD 120 is USD 5 and the price of a European put option with same strike and expiration is USD 25. What strategy exploits an arbitrage opportunity, if any?
    A. Short one put, short one unit of spot, buy one call, and buy six units box-spread.
    B. Buy one put, short one unit of spot, short one call, and buy four units of box-spread.
    C. Buy one put, buy one unit of spot, short one call, and short six units of box-spread.
    D. There are no arbitrage opportunities.

13. On March 13, 2008, William Tell, a fund manager for the Rossini fund, takes a short position in the March Treasury bond (T-bond) futures contract. He plans to deliver the cheapest-to-deliver Treasury bond with a coupon of 4.5% payable semiannually on May 15 and November 15 (182 days between), a conversion factor of 1.3256, and a face value of USD 100,000. The delivery date is Friday, March 15 (121 days after November 15 coupon payment date). The settlement price for the cheapest-to-deliver Treasury bond on March 13 is 68 2/32. Which of the following is the best estimate of the invoice price?
    A. USD 90,118.87
    B. USD 91,719.53
    C. USD 92,367.75
    D. USD 95,619.47

14. Nicholas is responsible for the asset and liability management of JerseyBeech Bank, a small retail bank with USD 300 million in interest-bearing assets that yield approximately 70 bp above LIBOR. The duration of the interest-bearing assets is 2.5 years. Due to the recent financial turmoil, the bank seeks to reduce potential negative impacts on earnings from adverse moves in interest rates. Thus, the bank decides to hedge 50% of its interest rate exposures using Treasury bond futures. Nicholas decides to use September T-bond futures that trade at 106-22 and will mature in three months; the cheapest-to-deliver bond associated with this contract is a 7-year, 10% coupon, with a current duration of 5 years. At the maturity of the futures contract, the duration of the bank's interest rate sensitive assets will not change; however, the duration of the cheapest-to-deliver bond will fall to 4.9. How many contracts should Nicholas buy or sell?
    A. Buy 703 contracts.
    B. Sell 703 contracts.
    C. Buy 717 contracts.
    D. Sell 717 contracts.

15. The yield curve is upward sloping, and a portfolio manager has a long position in 10-year Treasury Notes funded through overnight repurchase agreements. The risk manager is concerned with the risk that market rates may increase further and reduce the market value of the position. What hedge could be put on to reduce the position's exposure to rising rates?
    A. Enter into a 10-year pay fixed and receive floating interest rate swap.
    B. Enter into a 10-year receive fixed and pay floating interest rate swap.
    C. Establish a long position in 10-year Treasury Note futures.
    D. Buy a call option on 10-year Treasury Note futures.

16. The current price of stock ABC is USD 42 and the call option with a strike at USD 44 is trading at USD 3. Expiration is in one year. The put option with the same exercise price and same expiration date is priced at USD 2. Assume that the annual risk-free rate is 10% and that there is a risk-free bond paying the risk-free rate that can be shorted costlessly. There are no transaction costs. Which of the following trading strategies will result in arbitrage profits?
    A. Long position in both the call option and the stock, and short position in the put option and risk-free bond.
    B. Long position in both the call option and the put option, and short position in the stock and risk-free bond.
    C. Long position in both the call option and risk-free bond, and short position in the stock and the put option.
    D. Long position in both the put option and the risk-free bond, and short position in the stock and the call option.

17. Which of the following statements are correct about the early exercise of American options?

    I. It is never optimal to exercise an American call option on a non-dividend-paying stock before the expiration date.
    II. It can be optimal to exercise an American put option on a non-dividend-paying stock early.
    III. It can be optimal to exercise an American call option on a non-dividend-paying stock early.
    IV. It is never optimal to exercise an American put option on a non-dividend-paying stock before the expiration date.

    A. I and II
    B. I and IV
    C. II and III
    D. III and IV

18. Which one of the following four trading strategies limits the investor's upside potential and downside risk?
    A. A long position in a put combined with a long position in a stock.
    B. A short position in a put combined with a short position in a stock.
    C. Buying a call option on a stock with a certain strike price and selling a call option on the same stock with a higher strike price and the same expiration date.
    D. Buying a call and a put with the same strike price and expiration date.

19. The current spot price of cotton is USD 0.7409 per pound. The cost of storing and insuring cotton is USD 0.0042 per pound per month payable at the beginning of every month. The risk-free rate is 5%. A 3-month forward contract trades at USD 0.7415 per pound. If there is an arbitrage opportunity, how would you capitalize on it to make a profit? Assume there are no restrictions on short selling cotton.

    I. Short the futures contract
    II. Borrow at the risk-free rate
    III. Buy cotton at the spot price
    IV. Go long in the futures contract
    V. Invest at the risk-free rate
    VI. Sell cotton at the spot price

    A. There is no arbitrage opportunity here.
    B. The arbitrage opportunity involves I, II, and III.
    C. The arbitrage opportunity involves IV, V, and VI.
    D. The arbitrage opportunity involves II, IV, and VI.

20. If the lease rate of commodity A is less than the risk-free rate, what is the market structure of commodity A?
    A. Backwardation
    B. Contango
    C. Flat
    D. Inversion

21. Basis risk is a common problem faced by hedgers because the underlying and the hedging instrument may not always move in perfect correlation. Which of the following strategies has the least basis risk?
    A. Straddle strategy
    B. Hedging individual equities using index futures
    C. Stack and roll strategy
    D. Delta hedging strategy

22. A firm is going to buy 10,000 barrels of West Texas Crude Oil. It plans to hedge the purchase using the Brent Crude futures contract. The correlation between the spot and futures prices is 0.72. The volatility of the spot price is 0.35 per year. The volatility of the Brent Crude futures price is 0.27 per year. What is the hedge ratio for the firm?
    A. 0.5554
    B. 0.9333
    C. 1.2099
    D. 0.8198

23. The current value of the S&P 500 index is 1457, and each S&P futures contract is for delivery of USD 250 times the index. A long-only equity portfolio with market value of USD 300,100,000 has beta of 1.1. To reduce the portfolio beta to 0.75, how many S&P futures contracts should you sell?
    A. 618 contracts
    B. 288 contracts
    C. 574 contracts
    D. 906 contracts

24. A 3 month futures contract on an equity index is currently priced at USD 1000, the underlying index stocks are valued at USD 990 and pay dividends at a continuously-compounded rate of 2 percent and the current continuously compounded risk-free rate is 4 percent. The potential arbitrage profit per contract, given this set of data, is closest to:
    A. USD 10.00
    B. USD 7.50
    C. USD 5.00
    D. USD 1.50

25. It is June 2nd and a fund manager with USD 10 million invested in government bonds is concerned that interest rates will be highly volatile over the next three months. The manager decides to use the September Treasury bond futures contract to hedge the value of the portfolio. The current futures price is 95.0625. Each contract is for the delivery of USD 100,000 face value of bonds. The duration of the manager's bond portfolio in three months will be 7.8 years. The cheapest to deliver bond in the Treasury bond futures contract is expected to have a duration of 8.4 years at maturity of the contract. At the maturity of the Treasury bond futures contract, the duration of the underlying benchmark Treasury bond is 9 years. What position should the fund manager undertake to mitigate his interest rate risk exposure?
    A. Short 94 contracts
    B. Short 98 contracts
    C. Short 105 contracts
    D. Short 113 contracts

26. Consider an FRA (forward rate agreement) with the same maturity and compounding frequency as a Eurodollar futures contract. The FRA has a LIBOR underlying. Which of the following statements are true about the relationship between the forward rate and the futures rate?
    A. They should be exactly the same
    B. The forward rate is normally higher than the futures rate
    C. The forward rate is normally lower than the futures rate
    D. They have no fixed relationship

27. To hedge against future, unanticipated, and significant increases in borrowing rates, which of the following alternatives offers the greatest flexibility for the borrower?
    A. Fixed for floating swap
    B. Interest rate collar
    C. Interest rate floor
    D. Call swaption

28. According to put-call parity, buying a put option on a stock is equivalent to:
    A. buying a call option and buying the stock with funds borrowed at the risk-free rate.
    B. selling a call option and buying the stock with funds borrowed at the risk-free rate.
    C. buying a call option, selling the stock and investing the proceeds at the risk-free rate.
    D. selling a call option, selling the stock and investing the proceeds at the risk-free rate.

29. Your bank is an active player in the commodity market. The view of the economist of the bank is that inflation is expected to rise moderately in the near term and market volatility is expected to remain low. The traders are advised to undertake deals on the metals exchange to align your book to conform with the expectations of the economist of the bank. As risk manager, you are asked to monitor the positions of the traders to make sure that they have the exposures to inflation and market volatility sought by the bank. Which trader has taken an appropriate position among the traders you are monitoring?
    A. Trader A bought a call and a put, both with 90-days to expiration and with strike price equal to the existing spot level
    B. Trader B bought a put option with a down-and-in knock in feature
    C. Trader C bought a call option at the existing spot levels and sold a call at a higher strike price, both with 90-days to expiration
    D. Trader D sold a call and bought a put at the existing levels, both with 90-days to expiration

30. An investor sells a June 2008 call of ABC Limited with a strike price of USD 45 for USD 3 and buys a June 2008 call of ABC Limited with a strike price of USD 40 for USD 5. What is the name of this strategy and the maximum profit and loss the investor could incur?
    A. Bear Spread, Maximum Loss USD 2, Maximum Profit USD 3
    B. Bull Spread, Maximum Loss Unlimited, Maximum Profit USD 3
    C. Bear Spread, Maximum Loss USD 2, Maximum Profit Unlimited
    D. Bull Spread, Maximum Loss USD 2, Maximum Profit USD 3

31. Research and model projections indicate that a specific event is likely to move the CHF against the USD. While the direction of the move is highly uncertain, it is highly likely that magnitude of the move will be significant. Based on this information, which of the following strategies would provide the largest economic benefit?
    A. Long a call option on USD/CHF and short a put option on USD/CHF with the same strike price and expiration date
    B. Long a call option on USD/CHF and long a put option on USD/CHF with the same strike price and expiration date
    C. Short a call option on USD/CHF and long a put option on USD/CHF with the same strike price and expiration date
    D. Short a call option on USD/CHF and short a put option on USD/CHF with the same strike price and expiration date

32. Which of the following strategies creates a calendar spread?
    A. Sell a call option with a certain strike price and buy a longer maturity call option with the same strike price
    B. Buy a call option with a certain strike price and buy a longer maturity call option with the same strike price
    C. Sell a call option with a certain strike price and buy a shorter maturity call option with the same strike price
    D. Buy a call option with a certain strike price and sell a longer maturity call option with the same strike price

33. If the gold lease rate is higher than the risk-free rate, what is the market structure of the forward market for gold?
    A. Contango
    B. Backwardation
    C. Inversion
    D. Need more information to determine

34. Given the following explanation of supply and demand for commodity product A how would you best describe its forward price curve from June to December?

	Market description	Explanation
A.	Backwardation	Excess demand for A in early summer
B.	Backwardation	Supply is expected to decline after summer
C.	Contango	Excess demand for A in early summer
D.	Contango	Supply is expected to decline after summer

35. What is the annualized rate of return earned on a cash-and-carry trade entered into in March and closed out in June?
    A. 8.9%
    B. 9.8%
    C. 35.7%
    D. 39.1%

36. Which of the following trade(s) contain basis risk?

    I. Long 1,000 lots Nov 07 ICE Brent Oil contracts and short 1,000 lots Nov 07 NYMEX WTI Crude Oil contracts
    II. Long 1,000 lots Nov 07 ICE Brent Oil contracts and long 2,000 lots Nov 07 ICE Brent Oil at-the-money puts
    III. Long 1,000 lots Nov 07 ICE Brent Oil contracts and short 1,000 lots Dec 07 ICE Brent Oil contracts
    IV. Long 1,000 lots Nov 07 ICE Brent Oil contracts and short 1,000 lots Dec 07 NYMEX WTI Crude Oil contracts

    A. I and III
    B. II and IV
    C. III and IV
    D. I, III, and IV

37. Consider a 6-month futures contract on the S&P 500, and suppose the current value of the index is 1330. Suppose the dividend yield is 1.5% annually for the stocks underlying the index, and that the continuously compounded risk-free interest rate is 5.5% annually. What is the cost of carry for this futures contract?
    A. 4.0%
    B. –4.0%
    C. 2.0%
    D. –2.0%

38. You have entered into a currency swap in which you receive 4% per annum in yen and pay 6% per annum in dollars once a year. The principals in the two currencies are 100 million yen and 10 million dollar. The swap will last for another two years, and the current exchange rate is 115 yen for 1 dollar. Suppose that the annualized spot rates (with continuous compounding) are given as in the table below, what is the value of the swap to you in million dollars?

Professor's Note: According to the answer and the 115 yen per dollar exchange rate, the notional principal in yen should actually be 1 billion yen, not 100 million yen.

	1 Year	2 Year
Japan	2.00%	2.50%
United States	4.50%	4.75%

    A. –1.270
    B. –0.447
    C. 0.447
    D. 1.270

39. You are given the following information about an interest rate swap:
- 2-year term.
- Semi-annual payment.
- Fixed rate = 6%.
- Floating rate = LIBOR + 50 basis points.
- Notional principal = USD10 million.

Calculate the net coupon exchange for the first period if LIBOR is 5% at the beginning of the period and 5.5% at the end of the period.
A. Fixed rate payer pays USD 0
B. Fixed rate payer pays USD 25,000
C. Fixed rate payer pays USD 50,000
D. Fixed rate payer receives USD 25,000

40. Which of the following statements about American options is false?
A. American options can be exercised at any time until maturity.
B. American options are always worth at least as much as European options.
C. American options can easily be valued with Monte Carlo simulation.
D. American options can be valued with binomial trees.

41. Which of the following factors will not necessarily increase the price of a European call option on a dividend paying stock as this factor increases in value?
A. The risk free rate
B. The stock price
C. The time to expiration
D. The volatility of the stock price

42. An American investor holds a portfolio of French stocks. The market value of the portfolio is €10 million, with a beta of 1.35 relative to the CAC index. In November, the spot value of the CAC index is 4,750. The exchange rate is USD 1.25/€. The dividend yield, euro interest rates, and dollar interest rates are all equal to 4%. Which of the following option strategies would be most appropriate to protect the portfolio against a decline of the euro that week?

March Euro options (all prices in US dollars per €)

Strike	Call euro	Put euro
1.25	0.018	0.022

A. Buy calls with a premium of USD 180,000
B. Buy puts with a premium of USD 220,000
C. Sell calls with a premium of USD 180,000
D. Sell puts with a premium of USD 220,000

43. A portfolio manager wants to hedge his bond portfolio against changes in interest rates. He intends to buy a put option with a strike price below the portfolio's current price in order to protect against rising interest rates. He also wants to sell a call option with a strike price above the portfolio's current price in order to reduce the cost of buying the put option. What strategy is the manager using?
A. Bear spread
B. Strangle
C. Collar
D. Straddle

44. Imagine a stack-and-roll hedge of monthly commodity deliveries that you continue for the next five years. Assume the hedge ratio is adjusted to take into effect the mistiming of cash flows but is not adjusted for the basis risk of the hedge. In which of the following situations is your calendar basis risk likely to be greatest?
A. Stack and roll in the front month in oil futures.
B. Stack and roll in the 12-month contract in natural gas futures.
C. Stack and roll in the 3-year contract in gold futures.
D. All four situations will have the same basis risk.

45. Which of the following best describes what we would normally expect to see in a seasonal agricultural market like wheat? Assume "the harvest" is normal and not unusually big or unusually small. Now consider the following statements about the market.
I. Prices fall at the harvest and rise after the harvest.
II. Prices are constant on average across the year regardless of seasonality.
III. Prices rise at the harvest and fall afterwards.
IV. The market is in contango when the harvest comes in.
V. The market is in backwardation when the harvest comes in.
VI. If the market goes into contango, it is most likely to do so right before a new harvest.
VII. If the market goes into backwardation, it is most likely to do so right before a new harvest.

Now choose the letter that best describes which of the above statements is true.
A. I and IV are the only true statements.
B. I, IV, and VI are the only true statements.
C. III, V, and VII are the only true statements.
D. I, IV, and VII are the only true statements.

GARP FRM Practice Exam Answers

Financial Markets and Products

Question from the 2011 FRM practice exam.

1. **D** USD 1,710

Day	*Futures Price*	*Daily Gain (Loss)*	*Cumulative Gain (Loss)*	*Margin Account Balance*	*Margin Call*
June 1	497.30	(270)	(270)	1730	
June 2	492.70	(460)	(730)	1270	
June 3	484.20	(850)	(1580)	420	1580
June 4	471.70	(1250)	(2830)	750	1250
June 5	468.80	(290)	(3120)	1710	

(See Topic 24)

Question from the 2011 FRM practice exam.

2. **B** July 1 and July 2 only

Here is the complete history of the margin account and margin calls:

Day	*Futures Price*	*Daily Gain (Loss)*	*Cumulative Gain (Loss)*	*Margin Account Balance*	*Margin Call*
6/29/2010	18.62			6,000	0
6/30/2010	18.69	700	700	6,700	0
7/1/2010	18.03	–6,600	–5,900	100	5,900
7/2/2010	17.72	–3,100	–9,000	2,900	3,100
7/6/2010	18.00	2,800	–6,200	8,800	0
7/7/2010	17.70	–3,000	–9,200	5,800	0
7/8/2010	17.60	–1,000	–10,200	4,800	0

Margin calls happened on July 1 and July 2 only.

(See Topic 24)

Question from the 2011 FRM practice exam.

3. C Basis risk exists when futures and spot prices change by the same amount over time and converge at maturity of the futures contract.

Statement a is incorrect: as it is a correct statement: An Airline company hedging jet fuel with heating oil futures may face basis risk due to difference in the underlying assets.

Statement b is incorrect: as it is a correct statement: Optionalities left to the seller at maturity gives the seller flexibility resulting in the buyer of the contract facing basis risk.

Statement c is correct: as it is an incorrect statement: Basis risk exists when futures and spot prices do not change by the same amount over time and possibly will not converge at maturity of the futures contract.

Statement d is incorrect: as it is a correct statement: The magnitude of basis risk depends mainly on the degree of correlation between cash and futures prices. If the correlation is one then by definition there is no basis risk.

(See Topic 25)

Question from the 2011 FRM practice exam.

4. C Sell 167 futures contracts of S&P 500

The calculation is as follows: Two-thirds of the equity fund is worth USD 40 million.

The optimal hedge ratio is given by h = 0.89 * 0.51 / 0.48 = 0.945

The number of futures contracts is given by

N = 0.945 * 40,000,000 / (910 * 250) = 166.26 $\approx$ 167, round up to nearest integer.

(See Topic 25)

Question from the 2011 FRM practice exam.

5. C The inability of managers to forecast the price of the underlying is an argument for hedging but does not increase basis risk.

(See Topic 25)

Question from the 2011 FRM practice exam.

6. C Sell 205 futures contracts of S&P 500

The calculation is as follows:

The equity fund is worth USD 50 million.

The optimal hedge ratio is given by h = 0.92 * 0.55 / 0.45 = 1.124

The number of futures contracts is given by

N = 1.124 * 50,000,000 / (1,100 * 250) = 204.36 $\approx$ 205, round up to nearest integer.

(See Topic 25)

Question from the 2011 FRM practice exam.

7. **B** 40

With the given data, the no-arbitrage futures price should be:
$800e^{(0.025 - 0.03)*0.50} = 798$

Since the market price of the futures contract is lower than this price there is an arbitrage opportunity, the futures contract could be purchased and the index sold.

Arbitrage profit is 798 – 758 = 40

(See Topic 27)

Question from the 2011 FRM practice exam.

8. **B** Bond B

The cheapest to deliver bond on maturity is defined to be the one for which the adjusted spot price is the lowest.

Adjusted spot price = spot price / conversion factor. Computation of adjusted price is shown below for each of the bonds:

Bond A = 102.44 / 0.98 = 104.53%

Bond B = 106.59 / 1.03 = 103.49%

Bond C = 98.38 / 0.95 = 103.55%

So, bond B is the cheapest to deliver bond and option B is correct.

(See Topic 28)

Question from the 2011 FRM practice exam.

9. **B** Bond B

The cheapest to deliver bond on maturity is defined to be the one for which the adjusted spot price is the lowest.

Adjusted spot price = spot price / conversion factor. Computation of adjusted price is shown below for each of the bonds:

Bond A = 102.4 / 0.8 = 128%

Bond B = 100.4 / 1.5 = 67%

Bond C = 99.6 / 1.1 = 91%

So, bond B is the cheapest to deliver bond and option B is correct.

(See Topic 28)

Question from the 2011 FRM practice exam.

10. **C** USD 4 million

Fixed rate coupon = USD 300 million × 7.5% = USD 22.5 million

Value of the fixed payment = $B_{fix} = 22.5e^{(-0.07)} + 322.5e^{(-0.08 * 2)}$ = USD 295.80 million

Value of the floating payment = $B_{floating}$ = USD 300 million. Since the payment has just been made the value of the floating rate is equal to the notional amount.

Value of the swap = $B_{floating} - B_{fix}$ = USD 300 – USD 295.80 = USD 4.2 million

(See Topic 29)

Question from the 2011 FRM practice exam.

11. **C** 3.52%

Due to the fact that the American call option under consideration is on the stock which does not pay dividends, its value is equal to the European call option with the same parameters. Thus, we can apply put-call parity to determine the level of interest rate.

$C - P = S - Ke^{-rT}$

$0.46 - 2.25 = 22 - 24e^{-0.25r}$

$-23.79 = -24e^{-025r}$

$r = 3.52\%$

(See Topic 30)

Question from the 2011 FRM practice exam.

12. **A** Short one put, short one unit of spot, buy one call, and buy six units of box-spread.

The key concept here is the box-spread. A box-spread with strikes at USD 120 and USD 150, gives you a pay-off of USD 30 at expiration irrespective of the spot price.

Now recall the put call parity relation:

p + S = c + price of zero coupon bond with face value of strike redeeming at the maturity of the options

Since the strike is USD 120, price of a zero coupon bond with face value of USD 120 can be expressed as 4 units of box spread.

Strategy A is correct

Short one put: +25
Short one spot: +100
Buy one call: –5
Buy six box-spreads: –120
Net cash flow: 0

At expiry, if spot is greater than 120, call is exercised and if it is less than 120, put is exercised. In either case you end up buying one spot at 120. This can be used to close the short position. The six spreads will provide a cash flow of 6*30 = 180. The net profit is therefore = 180 – 120 = 60.

(See Topic 31)

Question from the 2010 FRM practice exam.

13. **B** USD 91,719.53

The invoice is based on a settlement price of 68 2/32 or 68.0625. The accrued interest is calculated on the basis of the number of days since the last coupon payment date, November 15, and the delivery date, March 15. That is 121. During the current six-month period between coupon payment dates, November 15 to May 15, there are 182 days. Thus the accrued interest on USD 100,000 face value of the bond is 121/182 * USD 100,000 * 0.045/2 = USD 1,495.88.

The invoice price is USD 100,000 * 0.680625 * 1.3256 + USD 1,495.88 = 91,719.53.

(See Topic 28)

Question from the 2010 FRM practice exam.

14. **D** Sell 717 contracts.

Exposure to hedge * Duration of assets to be hedged = 150M * 2.5 = 375M

Price of futures contract * Duration of futures contract = 106.6875% * 100,000 * 4.9 = 522,768.75

717 contracts = 375M / 0.52276875M

Since he is long in the asset, he should sell 717 contracts. The answer with 703 contracts comes from not using the duration at the maturity of the futures contract.

(See Topic 28)

Question from the 2010 FRM practice exam.

15. A Enter into a 10-year pay fixed and receive floating interest rate swap.

A. Correct. An increase in rates will increase the value of the hedge position and offset the loss in value from the bond position.
B. Incorrect. An increase in rates will decrease the value of the hedge position and add to the loss in value from the bond position.
C. Incorrect. An increase in rates will decrease the value of the futures position and add to the loss in value from the bond position.
D. Incorrect. An increase in rates (all else equal), will decrease the value of the call option and add to the loss in value from the bond position.

(See Topic 29)

Question from the 2010 FRM practice exam.

16. **C** Long position in both the call option and risk-free bond, and short position in the stock and the put option.

A. Incorrect. This would not yield arbitrage profit.
B. Incorrect. This would not yield arbitrage profit.
C. Correct. The put call parity relation is: stock + put = pv(strike) + call.
Therefore for no arbitrage opportunity the following relation should hold 42 + 2 = (44/1.10) + 3. But 44 > 43.
Therefore there is an arbitrage opportunity. The arbitrage profit is 44 – 43 = 1 by taking a long position in call and buying the risk-free bond and going short on the stock and the put.
D. Incorrect. This would not yield arbitrage profit.

(See Topic 30)

Question from the 2010 FRM practice exam.

17. A I and II

There are no advantages to exercising early if the investor plans to keep the stock for the remaining life of the call option, because the early exercise would sacrifice the interest that would be earned if the strike price is paid out later on expiration date after the early exercise, the investor may suffer the risk that the stock price will fall below the strike price as the stock pays no dividend, the early exercise will earn no income from the stock. So it is never optimal to exercise an American call option on a nondividend-paying stock before the expiration date. At any given time during its life, a put option should always be exercised early if it is sufficiently deep in the money. So it can be optimal to exercise an American put option on a non-dividend-paying stock early. As a result, answer A is correct.

(See Topic 30)

Question from the 2010 FRM practice exam.

18. C Buying a call option on a stock with a certain strike price and selling a call option on the same stock with a higher strike price and the same expiration date.

    A. Incorrect. Long position in a put combined with long position in a stock could limit only the downside risk.
    B. Incorrect. Short position in a put combined with short position in a stock could limit only the upside risk.
    C. Correct. Buying a call option on a stock with a certain strike price and selling a call option on the same stock with a higher strike price and the same expiration date could limit both the upside and downside risk.
    D. Incorrect. Buying a call and put with the same strike price and expiration date could limit only the downside risk.

    (See Topic 31)

Question from the 2010 FRM practice exam.

19. C The arbitrage opportunity involves IV, V, and VI.

    A. Incorrect. There exists an arbitrage opportunity on account of price differentials.
    B. Incorrect. As the futures price is lower than the observed price (future spot price), you need to long the futures and not short it.
    C. Correct. Such a strategy results in profit, as shown below.
       The future spot price is USD 0.7428: {0.7409 e ^ [0.05 * (3/12)]} + [0.0042 (1 + 0.05 / 12)^3 + 0.0042 (1 + 0.05 / 12)^2 + 0.0042 (1 + 0.05 / 12)] = 0.7301 + 0.0127 = USD 0.7428
       The futures price is USD 0.7415, which is lower than USD 0.7428. Hence you need to buy the futures, sell cotton spot and invest the funds in a risk-free bond so as to obtain a riskless profit of USD 0.0013 per pound.
    D. Incorrect. Borrowing and buying cotton spot does not result in a profit.

    (See Topic 33)

Question from the 2010 FRM practice exam.

20. **B** Contango

    Contango occurs when futures prices are higher than current spot, so in this case the risk-free rate is greater than the lease rate.

    Backwardation occurs when futures prices are less than spot, so in this case the lease rate is greater than risk-free rate. So, if the lease rate is less than the risk-free rate, the futures price is above the current spot price.

    (See Topic 33)

Question from the 2010 FRM practice exam.

21. A Straddle strategy

    A straddle involves buying a call and a put for the same underlying at a given strike price. There is no basis risk. The other strategies have basis risk.

    (See Topics 31 and 33)

Question from the 2009 FRM practice exam.

22. **B** 0.9333

$$N = 0.72 \times \left(\frac{0.35}{0.27}\right)$$

$$N = 0.9333$$

A. Incorrect. Inverts the spot volatility and the futures volatility.
C. Incorrect. Uses variances.
D. Incorrect. Uses square roots of the volatilities.

(See Topic 25)

Question from the 2009 FRM practice exam.

23. **B** 288 contracts

No of contracts = [(0.75 – 1.1) / 1] * [300,100,000 / {250 * 1,457}] = –288.36 → sell 288 contracts

A. Incorrect. –617.9135209 = –1 * (0.75) * (300100000 / (250 * 1457))
C. Incorrect. –561.74 = –1(0.75 / 1.1) * (300100000 / (250 * 1457))
D. Incorrect. –906.273164 = –1 * (1.1) * (300100000 / (250 * 1457))

(See Topic 25)

Question from the 2009 FRM practice exam.

24. **C** USD 5.00

According to the fundamental pricing relationship between spot assets and the associated futures, the futures price, to prevent arbitrage, should equal 990 × e^[(0.04 – 0.02) * 0.25] or 995. Hence, the futures contract is overvalued, indicating it should be sold and the index should be purchased for an arbitrage profit of USD 1000 – USD 995 = USD 5.

(See Topic 27)

Question from the 2009 FRM practice exam.

25. **B** Short 98 contracts

$$N = \left(\frac{10{,}000{,}000}{95{,}062.50}\right) \times \left(\frac{7.8}{8.4}\right)$$

$$N = 97.68 \text{ or } 98 \text{ contracts}$$

A. Incorrect. This is made up.
C. Incorrect. This leaves out the durations.
D. Incorrect. This inverts the durations.

(See Topic 28)

Question from the 2009 FRM practice exam.

26. **C** The forward rate is normally lower than the futures rate

As Eurodollar futures contract is marked to market and settled daily, normally forward rate is adjusted lower, so called convexity adjustment, by:

$$\text{Forward rate} = \text{Futures rate} - \frac{1}{2}\sigma^2 T_1 T_2$$

(See Topic 28)

Question from the 2009 FRM practice exam.

27. **D** Call swaption

The question focuses on flexible management of borrowing expenses. While a fixed for floating swap could reduce borrowing expenses, it is a long-term contractual commitment to exchange payments. If interest rates decline, the borrower may gross up to the agreed fixed rate. An interest rate collar is a combination of an interest rate floor and cap, i.e., it locks in the interest expenses within a tight range. Moreover, collars usually offer interest rate protection at one particular point of time unless several contracts with different maturities are exchanged. A call swaption gives the company the right to enter into a swap when the borrowing expenses exceed a certain reference rate. If the reference rate is below the borrowing expenses, the option is not exercised.

(See Topic 29)

Question from the 2009 FRM practice exam.

28. **C** Buying a call option, selling the stock and investing the proceeds at the risk-free rate.

Buying a call option, selling the stock and investing the proceeds at the risk-free rate. Put-call parity states $P = C - S + X\,e^{-RT}$

A. Incorrect. Buying a call option is correct, but the rest of the statement is incorrect.
B. Incorrect. The entire statement is incorrect.
D. Incorrect. Selling a call option is incorrect, but the rest of the statement is correct.

(See Topic 30)

Question from the 2009 FRM practice exam.

29. **C** Trader C bought a call option at the existing spot levels and sold a call at a higher strike price, both with 90-days to expiration.

The strategy popularly known as the bull spread will result in positive payoff when the spot rises. As inflation increases, spot levels in commodities are expected to rise. Selling a call at a higher level will reduce the cost of the strategy. Although it may limit the upside, but that would be in line with the view as only a moderate rise is expected in spot.

A. Incorrect. The strategy popularly known as a straddle is to be used when the view is that the volatility in the market will rise, and there is no directional view on the spot.
B. Incorrect. The above option will be suitable when the spot is expected to fall from the existing levels.
D. Incorrect. The payoff in this case is similar to a short position in the spot and would make sense when the underlying is expected to fall.

(See Topic 31)

Question from the 2009 FRM practice exam.

30. **D** Bull Spread, Maximum Loss USD 2, Maximum Profit USD 3

Buying a call option at lower stock price and selling call option at higher strike price is called a Bull Spread. Bear Spread is buying the call option at higher price and selling the call at lower strike price.

The cost of the strategy will be USD 3 – USD 5 = –USD 2.

The payoff, when the stock price $S_T \leq 40$ will be –USD 2 (the cost of strategy) as none of the options will be exercised.

The payoff, when the stock price $S_T \geq 45$, (as both options will be exercise) will be USD 5.

Since the cost of strategy is 2, the profit will be USD 5 – USD 2 = USD 3.

When stock price is USD 40 < S_T < USD 45, only the call option bought by the investor would be exercised, hence the payoff will be $S_T - 40$. Since the cost of strategy is –USD 3, the net profit will be $S_T - 42$, which would always be lower than USD 3 when the stock is less than 45.

(See Topic 31)

Question from the 2009 FRM practice exam.

31. **B** Long a call option on USD/CHF and long a put option on USD/CHF with the same strike price and expiration date

The question tests on understanding of a "straddle" strategy and its application on currency trading. A long straddle strategy involves buying (long) a call and put option with the same strike price and expiration date, and will benefit most when the underlying moves away from the current equlibrium.

A. Incorrect. It sells a put option while it should buy one put.
C. Incorrect. It sells a call option while it should buy one call.
D. Incorrect. It sells both the call and put option while it should buy both.

(See Topic 31)

Question from the 2009 FRM practice exam.

32. **A** Sell a call option with a certain strike price and buy a longer maturity call option with the same strike price

(See Topic 31)

Question from the 2009 FRM practice exam.

33. **B** Backwardation

A lease rate higher than the risk fee rate will force a negatively sloped forward curve, i.e. backwardation.

A. Incorrect. The forward price = spot * e^(risk-free rate – lease rate). If the lease rate is higher than the risk free rate, forwards will be lower than spot, implying contango.
C. Incorrect. The term inversion is used to describe yield curves, not commodity forwards.
D. Incorrect. There is enough information in the question to provide an answer.

(See Topic 33)

Question from the 2009 FRM practice exam.

34. A Backwardation; Excess demand for A in early summer

When further-term commodity forwards have lower price than near-term forwards, the market is said to be in 'backwardation'. Possible explanation can be seasonality of product A—excess demand in early summer causes June forwards to have higher price.

B. Incorrect. Market description is correct, but explanation is not—expected decline in supply should increase further-term commodity forward price.
C. Incorrect. Wrong market description of contango.
D. Incorrect. Wrong market description of contango.

(See Topic 33)

Question from the 2009 FRM practice exam.

35. C 35.7%

By formula $F_{0,T} = S_0e^{rT} + C$, where $F_{0,T}$ = June forward price, S_0 = March forward price, r = risk free interest rate, T = length of cash-and-carry, C = storage cost

Solving $5.90 = 5.35e^{r*3/12} + 0.05$

Solution is r = 35.7%

A. Incorrect. 8.9 = LN((5.9 – 0.05) / 5.35) (forgets to annualize the return)
B. Incorrect. 9.8= LN(5.9 / 5.35) (forgets to include the storage cost and to annualize the return)
D. Incorrect. 39.1= (12 / 3)LN(5.9 / 5.35) – 0.05 (forgets to include the storage cost)

(See Topic 33)

Question from the 2009 FRM practice exam.

36. D I, III, and IV

Basis risk is spread risk, which arises from trading the spread (long and short 2 positively correlated assets or same asset with different expiration).

I is spread trade in highly correlated asset with same expiration month.

II faces gamma and vega risk.

III is spread trade in trading the flattening of the forward curve.

IV is spread trade in trading 2 assets with different expiration date.

(See Topic 33)

Question from the 2008 FRM practice exam.

37. A 4.0%.

Professor's Note: The cost of carry is 5.5% – 1.5% = 4%.

(See Topic 27)

Question from the 2008 FRM practice exam.

38. **A** –1.270.

Calculation

(1) The value BY of yen-denominated bond:
$BY = 40\ e^{-(2\%)\ 1} + 1040\ e^{-(2.5\%)\ 2} = 1028.487$
(2) The value BD of dollar-denominated bond:
$BD = 0.6\ e^{-(4.5\%)\ 1} + 10.6\ e^{-(4.75\%)\ 2} = 10.213$
(3) The value CY of yen-denominated coupons:
$CY = 40\ e^{-(2\%)\ 1} + 40\ e^{-(2.5\%)\ 2} = 77.257$
(4) The value CD of dollar-denominated coupons:
$CD = 0.6\ e^{-(4.5\%)\ 1} + 0.6\ e^{-(4.75\%)\ 2} = 1.119$

A. Correct. Value = $B_Y / 115 - B_D$ = 1028.487/115 – 10.213 = –1.270.
B. Incorrect. It's derived by missing to account for the principals: 77.257/115 – 1.119 = –0.447
C. Incorrect. It mixes up the values of paying vs. receiving.
D. Incorrect. It mixes up the values of paying vs. receiving and does not account for the principals.

(See Topic 29)

Question from the 2008 FRM practice exam.

39. **B** Fixed rate payer pays USD 25,000.

A. Incorrect. The candidate incorrectly uses the LIBOR rate at the end of the period.
B. Correct. Fixed rate payer pays USD25,000. See BELOW for details.
C. Incorrect. The candidate forgets to add the 50 basis points to the beginning LIBOR rate.
D. Incorrect. The candidate is confused about the cash flow direction. A net positive payment is paid by the fixed rate payer, not receiving.

Computational Details for Numerical Answer:

- Fixed rate payer pays 6%, therefore (0.06 / 2) × 10 million = USD 300,000.
- Interest rate swaps have payments in arrears. Floating rate payer pays LIBOR rate at the beginning of period + 0.50%, i.e. 5 % + 0.50% = 5.5 %.
  Therefore the floating rate payment = (0.055 / 2) × 10 million = USD 275,000.
- The net payment of USD 25,000 is paid by the fixed rate payer.

(See Topic 29)

Question from the 2008 FRM practice exam.

40. C American options can easily be valued with Monte Carlo simulation

It is hard to value American options with Monte Carlo simulation, because it uses a prospective approach rather than a retrospective one.

A. Correct. American options can be exercise at any time.
B. Correct. American options can be exercise at any time vs. only at maturity for European option, which make American option more valuable.
C. Incorrect. It is very difficult to apply Monte Carlo retrospectively.
D. Correct. We can value American options at each node of the binomial tree as if it could be different exercise dates.

(See Topic 30)

Question from the 2008 FRM practice exam.

41. **C** The time to expiration.

A. Incorrect. An increase in the risk free rate will decrease PV(X) and necessarily increase the price of the European call.
B. Incorrect. An increase in the stock price will necessarily increase the price of the European call.
C. Correct. Because dividends paid before the expiration of the option might decrease the value of the stock price, it is possible that the value of the call option will decrease as the time to expiration is increased passed scheduled dividend payout dates.
D. Incorrect. An increase in the underlying stock price will necessarily increase the price of the European call.

(See Topic 30)

Question from the 2008 FRM practice exam.

42. **B** Buy puts with a premium of USD 220,000

A. Incorrect. This would not protect against a decline in the euro and would rather provide upside in case of appreciation of the euro.
B. Correct. This would protect against a decline in the euro and the premium would be USD.022 × €10 million = USD220,000.
C. Incorrect. This would not protect against a decline in the euro and would rather make the investor subject to (theoretically) unlimited losses (writing naked calls); the amount of premium is also incorrect and should be USD.018 × €10 million = USD180,000.
D. Incorrect. This would not protect against a decline in the euro and would rather protect against a decline in the US dollar; the amount of premium is also incorrect and should be USD.022 × €10 million = USD 220,000.

(See Topic 31)

Question from the 2008 FRM practice exam.

43. **C** Collar

A. Incorrect. The description is not for a bear spread. A bear spread is created by buying a nearby put and selling a more distant put. A bear spread can also be set up using calls.
B. Incorrect.
C. Correct. The description is for a collar strategy which limits changes in the portfolio value in either direction. In other words, a collar is defined around the current portfolio value.
D. Incorrect. The description is not for a straddle. A straddle is created by buying a put and a call at the same strike price and expiration to take advantage of significant portfolio moves in either direction.

(See Topic 31)

Question from the 2008 FRM practice exam.

44. **A** Stack and roll in the front month in oil futures.

The oil term structure is highly volatile at the short end, making a front-month stack-and-roll hedge heavily exposed to basis fluctuations. In natural gas, much of the movement occurs at the front end, as well, so the 12-month contract won't move as much. In gold, the term structure rarely moves much at all and won't begin to compare with oil and gas.

(See Topic 33)

Question from the 2008 FRM practice exam.

45. **D** I, IV, and VII are the only true statements

The new harvest 'resets' the storage market. For a while, consumption and production occur directly from the new harvest, and prices are low. Prices begin to rise as storage begins to occur. As the next harvest approaches, inventory may get tight, sending the market into backwardation.

(See Topic 33)

Book 3 Formulas

Financial Markets and Products

call option payoff: $C_T = \max(0, S_T - X)$

put option payoff: $P_T = \max(0, X - S_T)$

forward contract payoff: payoff = $S_T - K$

where:
S_T = spot price at maturity
K = delivery price

basis = spot price of asset being hedged – futures price of contract used in hedge

hedge ratio: $HR = \rho_{S,F} \dfrac{\sigma_S}{\sigma_F}$

beta: $\dfrac{Cov_{S,F}}{\sigma_F^{\,2}} = \beta_{S,F}$

correlation: $\rho = \dfrac{Cov_{S,F}}{\sigma_S \sigma_F}$

hedging with stock index futures:

$$\text{number of contracts} = \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{value of futures contract}}\right)$$

$$= \beta_{\text{portfolio}} \times \left(\frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}}\right)$$

adjusting the portfolio beta: number of contracts $= (\beta^* - \beta)\dfrac{P}{A}$

discrete compounding: $FV = A\left(1 + \dfrac{R}{m}\right)^{m \times n}$

continuous compounding: $FV = Ae^{R \times n}$

forward rate agreement: cash flow (if receiving R_K) $= L \times (R_K - R) \times (T_2 - T_1)$
cash flow (if paying R_K) $= L \times (R - R_K) \times (T_2 - T_1)$

forward price: $F_0 = S_0 e^{rT}$

forward price with carrying costs: $F_0 = (S_0 - I)\, e^{rT}$

forward price when the underlying asset pays a dividend: $F_0 = S_0 e^{(r-q)T}$

$$\text{accured interest} = \text{coupon} \times \frac{\text{\# of days from last coupon to the settlement date}}{\text{\# of days in coupon period}}$$

cash price of a bond: cash price = quoted price + accrued interest

annual rate on a T-Bill: T-bill discount rate = $\dfrac{360}{n}(100 - Y)$

cheapest-to-deliver bond: quoted bond price – (QFP × CF)

Eurodollar futures price = $10,000[100 – (0.25)(100 – Z)]

convexity adjustment:

actual forward rate = forward rate implied by futures – $(0.5 \times \sigma^2 \times t_1 \times t_2)$

duration-based hedge ratio: $N = -\dfrac{P \times D_P}{F \times D_F}$

forward rate between T_1 and T_2: $R_{forward} = R_2 + (R_2 - R_1)\,\dfrac{T_1}{T_2 - T_1}$

put-call parity:

$S = c - p + Xe^{-rT}$
$p = c - S + Xe^{-rT}$
$c = S + p - Xe^{-rT}$
$Xe^{-rT} = S + p - c$

lower and upper bounds for options:

Option	*Minimum Value*	*Maximum Value*
European call	$c \geq \max(0, S_0 - Xe^{-rT})$	S_0
American call	$C \geq \max(0, S_0 - Xe^{-rT})$	S_0
European put	$p \geq \max(0, Xe^{-rT} - S_0)$	Xe^{-rT}
American put	$P \geq \max(0, X - S_0)$	X

bull call spread: profit = $\max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$

bear put spread: profit $= \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$

butterfly spread with calls:

$$\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

straddle: $\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$

strangle: $\text{profit} = \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$

variance of the basis: $\sigma^2_{S(t)-F(t)} = \sigma^2_{S(t)} + \sigma^2_{F(t)} - 2\sigma_{S(t)}\sigma_{F(t)}\rho_{S,F}$

$$\text{hedge effectiveness} = 1 - \frac{\sigma^2_{S(t)-F(t)}}{\sigma^2_{S(t)}}$$

pricing a commodity forward with a lease payment: $F_{0,T} = S_0 e^{(r - \delta_l)T}$

commodity forward pricing with storage costs: $F_{0,T} = S_0 e^{(r+\lambda)T}$

commodity forward pricing with convenience yield: $F_{0,T} = S_0 e^{(r-c)T}$

interest rate parity: $\text{forward} = \text{spot}\left[\frac{(1 + r_{DC})}{(1 + r_{FC})}\right]^T$

$$\text{forward} = \text{spot} \times e^{(r_{DC} - r_{FC})T}$$

nominal interest rate:

exact methodology: $(1 + r) = (1 + \text{real } r)[1 + E(i)]$

linear approximation: $r \approx \text{real} + E(i)$

original-issue discount (OID) = face value – offering price

dollar default rate:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{(\text{cumulative dollar value of all issuance}) \times (\text{weighted average \# of years outstanding})}$$

Using the Cumulative Z-Table

Probability Example

Assume that the annual earnings per share (EPS) for a large sample of firms is normally distributed with a mean of \$5.00 and a standard deviation of \$1.50. What is the approximate probability of an observed EPS value falling between \$3.00 and \$7.25?

If EPS = x = \$7.25, then $z = (x - \mu)/\sigma = (\$7.25 - \$5.00)/\$1.50 = +1.50$

If EPS = x = \$3.00, then $z = (x - \mu)/\sigma = (\$3.00 - \$5.00)/\$1.50 = -1.33$

For z-value of 1.50: Use the row headed 1.5 and the column headed 0 to find the value 0.9332. This represents the area under the curve to the left of the critical value 1.50.

For z-value of –1.33: Use the row headed 1.3 and the column headed 3 to find the value 0.9082. This represents the area under the curve to the left of the critical value +1.33. The area to the left of –1.33 is 1 – 0.9082 = 0.0918.

The area between these critical values is 0.9332 – 0.0918 = 0.8414, or 84.14%.

Hypothesis Testing – One-Tailed Test Example

A sample of a stock's returns on 36 non-consecutive days results in a mean return of 2.0%. Assume the population standard deviation is 20.0%. Can we say with 95% confidence that the mean return is greater than 0%?

H_0: $\mu \leq 0.0\%$, H_A: $\mu > 0.0\%$. The test statistic = z-statistic = $\dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ = (2.0 – 0.0) / (20.0 / 6) = 0.60.

The significance level = 1.0 – 0.95 = 0.05, or 5%.

Since this is a one-tailed test with an alpha of 0.05, we need to find the value 0.95 in the cumulative z-table. The closest value is 0.9505, with a corresponding critical z-value of 1.65. Since the test statistic is less than the critical value, we fail to reject H_0.

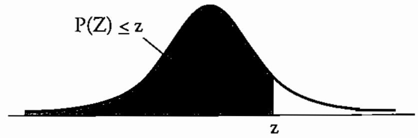
Hypothesis Testing – Two-Tailed Test Example

Using the same assumptions as before, suppose that the analyst now wants to determine if he can say with 99% confidence that the stock's return is not equal to 0.0%.

H_0: $\mu = 0.0\%$, H_A: $\mu \neq 0.0\%$. The test statistic (z-value) = (2.0 – 0.0) / (20.0 / 6) = 0.60. The significance level = 1.0 – 0.99 = 0.01, or 1%.

Since this is a two-tailed test with an alpha of 0.01, there is a 0.005 rejection region in both tails. Thus, we need to find the value 0.995 (1.0 – 0.005) in the table. The closest value is 0.9951, which corresponds to a critical z-value of 2.58. Since the test statistic is less than the critical value, we fail to reject H_0 and conclude that the stock's return equals 0.0%.

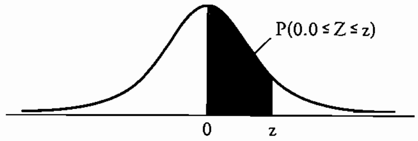
CUMULATIVE *Z*-TABLE



$P(Z \leq z) = N(z)$ for $z \geq 0$
$P(Z \leq -z) = 1 - N(z)$

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.937	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.983	0.9834	0.9838	0.9842	0.9846	0.985	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.989
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.994	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

ALTERNATIVE Z-TABLE



$P(Z \leq z) = N(z)$ for $z \geq 0$
$P(Z \leq -z) = 1 - N(z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3356	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4939	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

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